1 D.C. Circuits

1.1 : Electrical Circuit Elements (R, L and C)

Q.1 Write a note on circuit elements R, L and C, specifying voltage and current relations for them.
[JNTU : Part B, Dec.-11, 16, Marks 5]

Ans. :

1. **Resistance (R) :** The resistance is opposition to the flow of current. It is denoted as R and its symbol is shown in the Fig. Q.1.1.

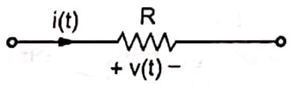

Fig. Q.1.1

- The factors affecting resistance of a material are,

i) Length (l) ii) Area of cross-section (a)

iii) The resistivity of the material (ρ)

- The resistance is given by,

$$R = \frac{\rho l}{a}\ \Omega$$

- It is measured in ohms.
- It's voltage and current relations are,

$$i(t) = \frac{v(t)}{R} \quad \text{and} \quad v(t) = i(t)\,R$$

2. **Inductance (L) :** The property of a coil which opposes any change in the current passing through it is called an inductance. It is denoted as L and its symbol is shown in the Fig. Q.1.2.

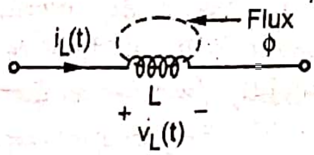

Fig. Q.1.2

- It stores energy in the form of an electromagnetic field.
- It is measured in henries (H).
- It is defined as total flux linking with the coil divided by the current producing the flux.

$$\therefore \quad L = \frac{N\phi}{i} \qquad \text{... N = Number of turns of coil}$$

- It's voltage and current relations are,

$$\therefore \quad i_L(t) = \frac{1}{L}\int_0^t v(t)dt + i_L(0),\ V_L(t) = L\frac{di_L(t)}{dt}$$

where $i_L(0)$ is initial current through L.

3. **Capacitance (C) :** The property of a capacitor to store an electrical energy in the form of an electrostatic field when a potential difference is applied across it is called capacitance.

- It is denoted as C and its symbol is shown in the Fig. Q.1.3.

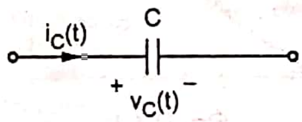

Fig. Q.1.3

- It is measured in farads (F).
- The charge acquired by a capacitor is proportional to the voltage applied and the constant of proportionality is called capacitance (C).

$$\therefore \quad C = \frac{Q}{V} \quad \text{or} \quad Q = CV$$

- It's voltage and current relations are,

$$\therefore \quad i_C(t) = C\frac{dv_C(t)}{dt} \quad \text{and} \quad v_C(t) = \frac{1}{C}\int i_C(t)dt + v_C(0)$$

where $v_C(0)$ is initial voltage on capacitor.

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Q.2 Write the V-I relations for R, L and C elements. [JNTU : Part-A, Aug.-18, , Marks 2]

Ans. : Refer answer of Q.1.

Q.3 Mention the factors affecting the resistance of a material. [JNTU : Part A : May-04, 08, 13, Dec.-05, 07, 09, Marks 3]

Ans. : The various factors affecting the resistance of a material are,

1) Directly proportional to length of the material.
2) Inversely proportional to **cross-sectional area** of the material.
3) Type and nature of the material, whether it is conductor, insulator or semiconductor.
4) Temperature.

Q.4 Draw the V-I characteristics of resistor. [JNTU : Part A, May-17, Marks 2]

Ans. : For resistance R, the relation between V and I is given by V = IR. Thus it is a linear relation and the V-I characteristics is straight line as shown in the Fig. Q.4.1.

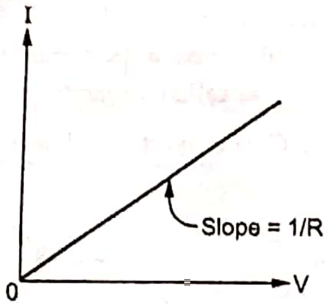

Fig. Q.4.1

Important Points to Remember

- Any arrangement of the various electrical energy sources along with the different circuit elements is called an **electrical network.**
- A part of the network which connects the various points of the network with one another is called a **branch.**
- A point where three or more branches meet is called a **junction point.**
- A point at which two or more branches meet is called **node.** The junction points are also the nodes of the network.
- Mesh (or Loop) is a set of branches forming a closed path in a network in such a way that if one branch is removed then remaining branches do not form a closed path.

1.2 : Voltage and Current Sources

Q.5 Discuss the different types of voltage and current sources used in electrical circuits. [JNTU : Part B, May-03,06, Dec.-07,18, Marks 5]

OR Define independent and dependent sources. [JNTU : Part A, Aug.-17, April-18, Marks 2]

Ans. : • There are two types of energy sources : voltage source and current source.

- These are further classified as independent sources and dependent sources.

1. **Independent voltage source :** The energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals is called **independent ideal voltage source.**

- But practically it has small internal resistance in series with it due to which voltage across its terminals decreases slightly as current increases. This is shown in the Fig. Q.5.1.

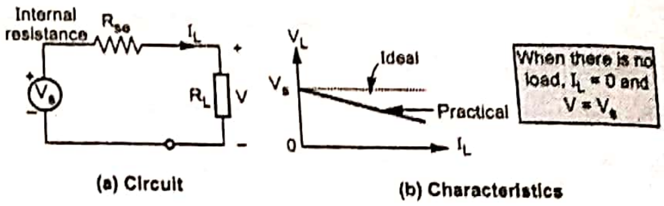

Fig. Q.5.1 Practical voltage source

- For ideal voltage source the value of the internal resistance is zero.

2. **Independent current source :** The source which gives constant current at its terminals irrespective of the voltage appearing across its terminals is called **ideal current source.**

- But practically it has high internal resistance connected in parallel with it. Due to this current supplied by it slightly deceases as the voltage at its teminal increases. This is shown in the Fig. Q.5.2.

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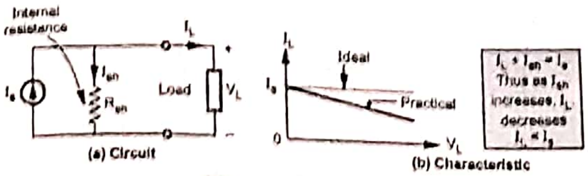


Fig. Q.5.2 Practical current source

- For ideal current source the internal resistance is infinite.

3. **Dependent sources :** The sources whose value depends on voltage or current, present somewhere else in the same circuit are called dependent sources.

- Such sources are indicated by diamond and further classified as,

i) **Voltage dependent voltage source :** It produces a voltage as a function of voltages elsewhere in the given circuit. This is called **VDVS.** It is shown in the Fig. Q.5.3 (a).

ii) **Current dependent current source :** It produces a current as a function of currents elsewhere in the given circuit. This is called **CDCS.** It is shown in the Fig. Q.5.3 (b).

iii) **Current dependent voltage source :** It produces a voltage as a function of current elsewhere in the given circuit. This is called **CDVS.** It is shown in the Fig. Q.5.3 (c).

iv) **Voltage dependent current source :** It produces a current as a function of voltage elsewhere in the given circuit. This is called **VDCS.** It is shown in the Fig. Q.5.3 (d).

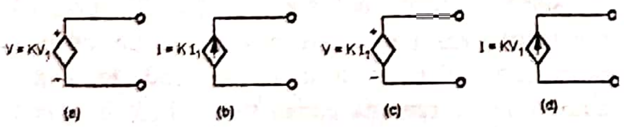


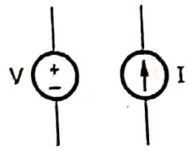
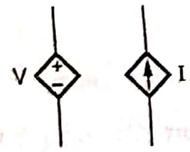
Fig. Q.5.3

- K is constant and V_1 and I_1 are the voltage and current respectively, present elsewhere in the given circuit. The dependent sources are also known as **controlled sources.**

Q.6 Differentiate between Independent and dependent sources.

[JNTU : Part B, May-19, Marks 5]

Ans. :

Sr. No.	Independent sources	Dependent sources
1.	It is the source whose value does not depend on any other quantity in the circuit. It maintains its value constant at all the times.	It is the source whose output value depends upon the voltage or current present elsewhere in the circuit.
2.	The two types are voltage source and current source.	The four types are voltage dependent voltage source (VDVS), voltage dependent current source (VDCS), current dependent voltage source (CDVS), current dependent current source (CDCS).
3.	Symbol is, 	Symbol is, 
4.	The strength of voltage or current is not changed by any variation in the connected network.	The strength of voltage or current gets changed by the variation in the connected network.

Important Points to Remember

- **Terminal voltage :** If a voltage source has finite resistance R_{se} and the current driven through load is I_L then the voltage available across the load is called terminal voltage of the source denoted as V_t. Mathematically it is given by, $V_t = V - I_L R_{se}$, as shown in the Fig. 1.1.

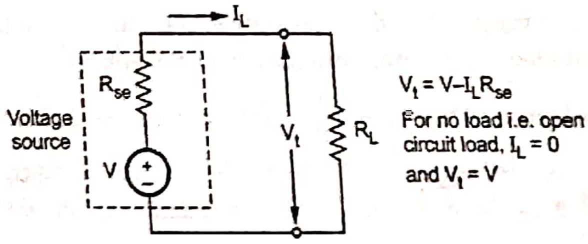


Fig. 1.1

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Q.7 A certain voltage source has a terminal voltage of 120 V when the load current is 1 A. When the load current is 2 A, the terminal voltage is 100 V. Calculate the internal resistance of the voltage source, open circuit voltage and short circuit current.

[JNTU : Part B, May-17, Marks 5]

Ans. :

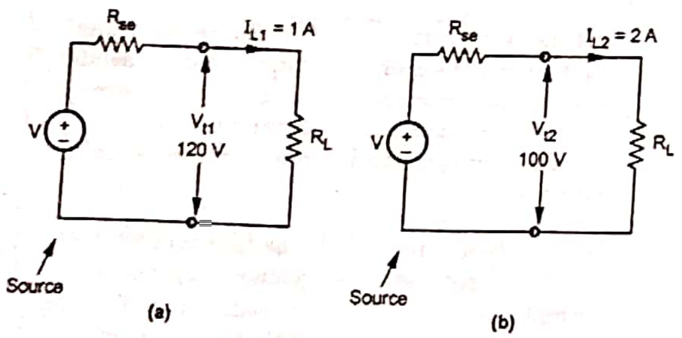

Fig. Q.7.1

$$V_t = V - I_L R_{se}$$

$\therefore$ $120 = V - 1 \times R_{se}$

and $100 = V - 2R_{se}$

Solving, $V = 140$ V,

$R_{se} = 20\ \Omega$

Open circuit voltage = V = 140 V

For short circuit, $R_L = 0\ \Omega$

$\therefore$ Short circuit current $= \dfrac{V}{R_{se}} = 7$ A

1.3 : Ohm's Law and Limitations

Q.8 State Ohm's law and its limitations.

[JNTU : Part A, May-06, 13,17, Dec.09,11,12,16, April-18, Marks 3]

Ans. : • The Ohm's law gives relationship between the potential difference (V), the current (I) and the resistance (R) of a d.c. circuit.

• It states that, the current flowing through the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provided the temperature remains constant.

• Mathematically, $I \propto \dfrac{V}{R}$ Where I is the current flowing in amperes, the V is the voltage applied and R is the resistance of the conductor, as shown in the Fig. Q.8.1.

Then $I = \dfrac{V}{R}$

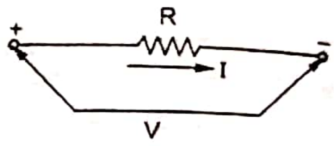

Fig. Q.8.1 Ohm's law

• The unit of potential difference is defined in such a way that the constant of proportionality is unity.

• Ohm's Law is, $I = \dfrac{V}{R}$ amperes or $V = IR$ volts

or $\dfrac{V}{I}$ = Constant = R ohms

• The Ohm's law can be defined as, the ratio of potential difference (V) between any two points of a conductor to the current (I) flowing between them is constant, provided that the temperature of the conductor remains constant.

• **The limitations of the Ohm's law are,**

1. It is not applicable to the nonlinear devices such as diodes, zener diodes, voltage regulators etc.
2. It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is given by,

   $V = k\, I^m$ where k, m are constants.
3. It is not applicable to electrolytes.
4. It is not applicable to discharge lamps and vacuum tubes.
5. It is valid only at constant temperatures.

1.4 : Series Circuit of Resistances

Important Points to Remember

• A series circuit is one in which several resistances are connected one after the other. Such connection is also called **end to end** connection or **cascade connection**. There is only one path for the flow of current.

• The total or **equivalent resistance** of the series circuit is arithmetic sum of the resistances connected in series.

For n resistances in series,

$$R_{eq} = R_1 + R_2 + R_3 + \ldots\ldots + R_n$$

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Q.9 State the characteristics of series circuit.

Ans. : 1) The same current flows through each resistance.

2) The supply voltage V is the sum of the individual voltage drops across the resistances i.e. $V = V_1 + V_2 + ... + V_n$

3) The equivalent resistance is equal to the sum of the individual resistances.

4) The equivalent resistance is the largest of all the individual resistances.

i.e. $R_{eq} > R_1,\ R_{eq} > R_2,\ .....\ R_{eq} > R_n$

1.5 : Parallel Circuit of Resistances

Important Points to Remember

- The **parallel circuit** is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point.

In general if 'n' resistances are connected in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$

Q.10 State the characteristics of parallel circuit.

Ans. : 1) The same potential difference gets across all the resistances in parallel.

2) The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of all the individual currents.

$$I = I_1 + I_2 + I_3 + ... + I_n$$

3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.

4) The equivalent resistance is the smallest of all the resistances.

$$R_{eq} < R_1,\ R_{eq} < R_2,\ .....,\ R_{eq} < R_n$$

5) The equivalent conductance is the arithmetic addition of the individual conductances.

- **The equivalent resistance is smaller than the smallest of all the resistances connected in parallel.**

Q.11 Find the equivalent resistance across the terminals A-B as shown in Fig. Q.11.1.

[JNTU : Part B, Aug.-17, Marks 5]

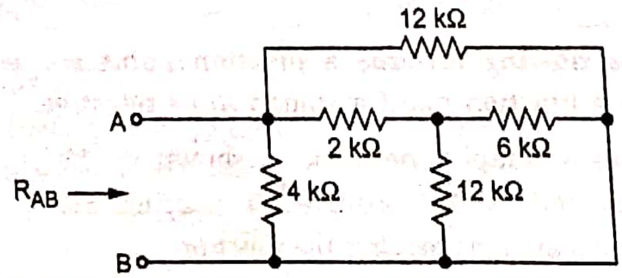

Fig. Q.11.1

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Ans. : The circuit can be rearranged as shown in the Fig. Q.11.1 (a).

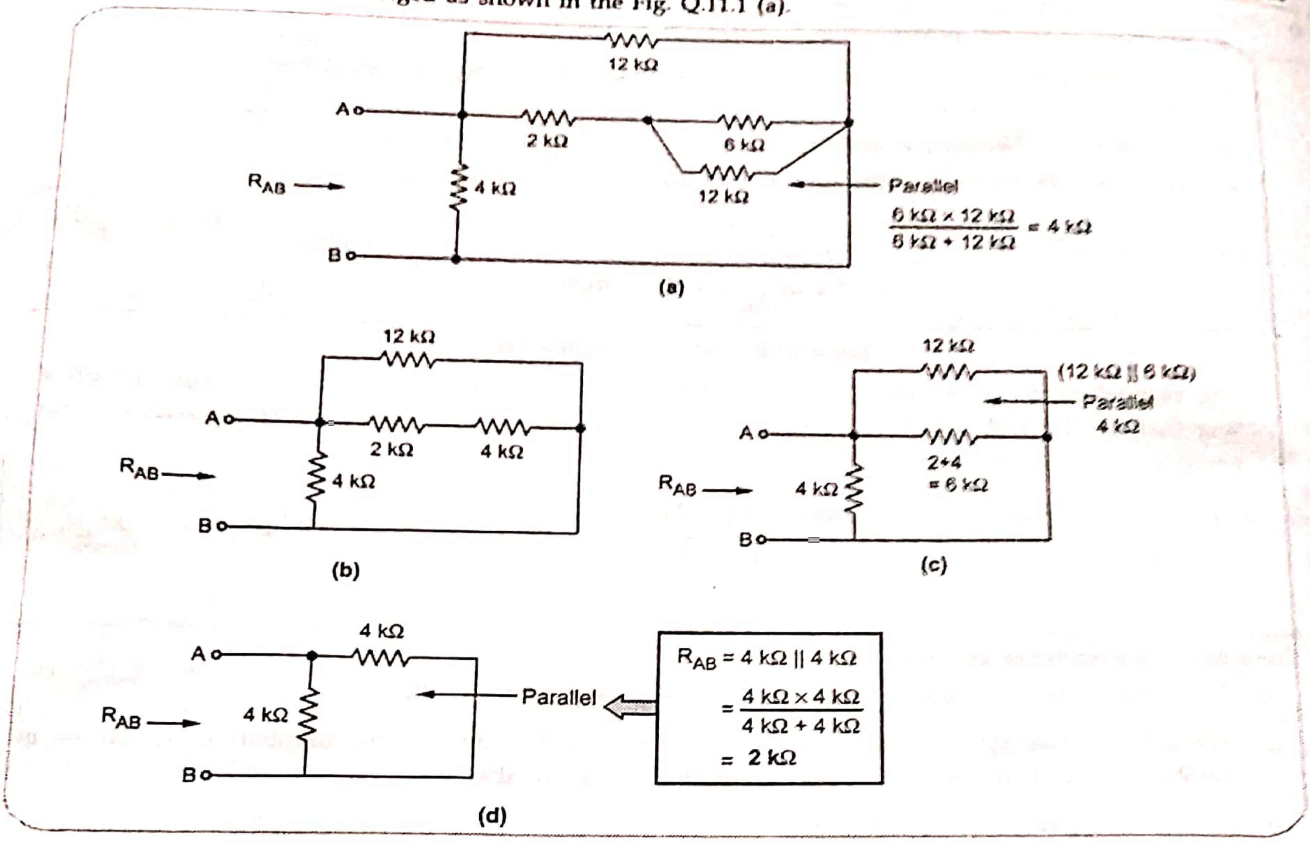

Fig. Q.11.1

1.6 : Kirchhoff's Laws

Q.12 State and explain Kichhoff's laws. State its applications and limitations.

[JNTU : Part B, May-04, 06, 09, 19, Dec.-05, 11, 12, Aug.-16,18 Marks 5]

Ans. : There are two Kirchhoff's laws.

1. Kirchhoff's Current Law (KCL)

- **The total current flowing towards a junction point is equal to the total current flowing away from that junction point.**
- Another way to state the law is,

The algebraic sum of all the current meeting at a junction point is always zero.

$$\sum I \text{ at junction point } = 0$$

Sign convention : Currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to be negative.

- Consider a junction point in a complex network as shown in the Fig. Q.12.1. The currents I_1 and I_2 are positive as entering the junction while I_3 and I_4 are negative as leaving the junction.
- Applying KCL, $\sum I$ at junction O = 0

$$I_1 + I_2 - I_3 - I_4 = 0 \quad \text{i.e.} \quad I_1 + I_2 = I_3 + I_4$$

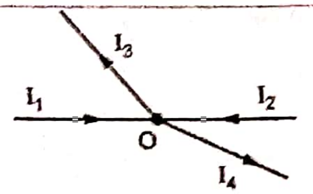

Fig. Q.12.1 Junction point

2. Kirchhoff's Voltage Law (KVL)

"In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.f.s in the path"

In other words, "the algebraic sum of all the branch voltages, around any closed path or closed loop is always zero."

$\therefore$ Around a closed path $\sum V = 0$

- The law states that if one starts at a certain point of a closed path and goes on tracing and noting all the potential changes (either drops or rises), in any one particular direction, till the starting point is reached again, he must be at the same potential with which he started tracing a closed path.
- Sum of all the potential rises must be equal to sum of all the potential drops while tracing any closed path of the circuit. The total change in potential along a closed path is always zero.

Applications :

- Kirchhoff's laws are used in the analysis of electrical networks, in mesh analysis, in node analysis, in the analysis various electronic ciruits etc.

Limitations :

1. Applicable to only lumped element model of the circuit.
2. Not applicable to high frequency circuits.
3. Not applicable if fluctuating magnetic field is linking with the closed looop.
4. Valid only if the total electric charge remains constant in the region for which the laws are to be applied.

Important Points to Remember

Sign conventions for KCL

- For KCL the currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to. be negative.

Sign conventions for KVL

- The current always flows from higher potential to lower potential. Hence the polarity of the voltage drop across the resistance along the current direction is to be marked as positive (+) to negative (–).
- While tracing a closed path for KVL, if we go from – ve marked terminal to + ve marked terminal, that voltage must be taken as positive. This is called **potential rise.**
- While tracing a closed path, if we go from +ve marked terminal to – ve marked terminal, that voltage must be taken as negative. This is called **potential drop.**
- The sum of all such potetial rises and potential drops must be zero for a closed loop under consideration, according to KVL.
- If there is current source in the network then complete the current distribution considering the current source. But while applying KVL, the **loops should not be considered involving current source. The loop equations must be written to those loops which do not include any current source.** This is because drop across current source is unknown.

Q.13 For the circuit shown in Fig. Q.13.1 find the current flowing in all the branches.

[JNTU : Part B, May-17, Marks 5]

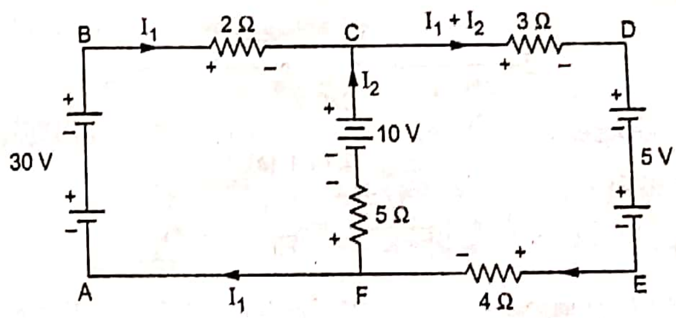

Fig. Q.13.1

Ans. : Apply KVL to the loops BCFAB and CDEFC

$$-2I_1 - 10 + 5I_2 + 30 = 0 \quad \text{i.e.}$$

$$-3(I_1 + I_2) - 5 - 4(I_1 + I_2) - 5I_2 + 10 = 0 \qquad ...(1)$$

$$-7I_1 - 12I_2 = -5 \qquad ...(2)$$

Solving,

$I_1 = 4.4915$ A, $I_2 = -2.203$ A, $I_1 + I_2 = 2.288$ A

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Hence currents in various branches are,

Branch	AB, BC, AF	CF	CD, DE, EF
Current	4.4915 A	– 2.203 A ↑ i.e. 2.203 A ↓	2.288 A

Q.14 In the circuit shown in below Fig. Q.14.1. Find the current in the each resistance.

☞ [JNTU [H] : Part B, Dec.-12, Marks 5]

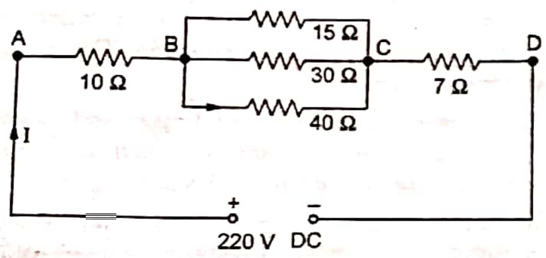

Fig. Q.14.1

Ans. : The resistances 15 Ω, 30 Ω, 40 Ω are in parallel.

$$\therefore \quad \frac{1}{R'} = \frac{1}{15}+\frac{1}{30}+\frac{1}{40} \quad \text{i.e} \quad R' = 8\ \Omega$$

The circuit becomes as shown in the Fig. Q.14.1 (a).

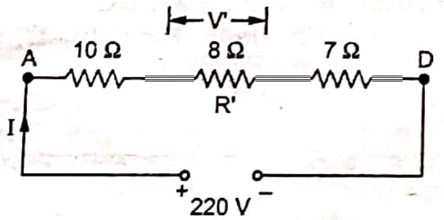

Fig. Q.14.1 (a)

$$I = \frac{V}{10+8+7} = \frac{220}{25} = 8.8\ A$$

∴ Drop across

$$R' = V' = R' \times I$$

$$= 8 \times 8.8 = 70.4\ V$$

But R′ is parallel combination of 15 Ω, 30 Ω, 40 Ω hence drop across all of them is same as V′ = 70.4 V.

$$\therefore \quad I_{15} = \frac{V'}{15} = 4.693\ A,$$

$$I_{30} = \frac{V'}{30} = 2.3467\ A,$$

$$I_{40} = \frac{V'}{40} = 1.76\ A$$

While $I_{10} = I_7 = I = 8.8$ A

Cross check that $I = I_{15} + I_{30} + I_{40}$

Q.15 For the circuit shown in Fig. Q.15.1. Obtain voltage between points X and Y.

☞ [VTU : Aug.-99, July-09,15,17, Marks 6]

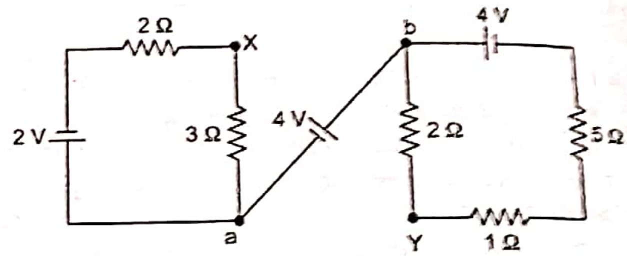

Fig. Q.15.1

Ans. : The branch currents are shown in the Fig. Q.15.1 (a). Apply KVL to the loop cXaac,

$$+2I_1 + 3I_1 - 2 = 0 \quad \text{i.e.} \quad I_1 = 0.4\ A$$

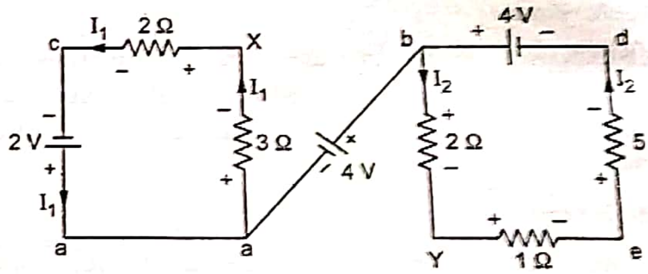

Fig. Q.15.1 (a)

Apply KVL to the loop bdeYb,

$$-4 + 5I_2 + I_2 + 2I_2 = 0$$

i.e. $I_2 = 0.5$ A.

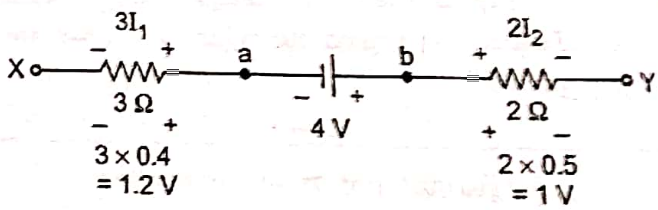

Fig. Q.15.1 (b)

To find V_{XY}, consider path XabY and write the voltage drops as shown in the Fig. Q.15.1 (b) across various elements. Adding algebraically the various voltages,

$$V_{XY} = 4 + 1.2 - 1$$

$= 4.2$ V **with X negative w.r.t. Y.**

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1.7 : Electrical Power

Q.16 Define electrical power and state its unit.

Ans. :

- *The rate at which electrical work is done in an electrical circuit is called an* **electrical power.** The unit of an electrical power is watt.

$$\text{Electrical power } P = \frac{\text{Electrical work}}{\text{Time}} = \frac{W}{t} = \frac{VIt}{t}$$

$$\therefore \quad P = VI \quad \text{J/sec i.e. watts}$$

- Thus power consumed in an electric circuit is 1 watt if the potential difference of 1 volt applied across the circuit causes 1 ampere current to flow through it.

Remember, 1 watt = 1 joule/sec

- According to Ohm's law, V = IR or I = V/R. Using this, power can be expressed as,

$$P = VI = I^2R = \frac{V^2}{R} \quad \text{where R = Resistance in } \Omega$$

- Practically the unit watt is very small hence large unit like kilowatt (kW) or magawatt (MW) is used.

$$1 \text{ kW} = 1000 \text{ W and } 1 \text{ MW} = 10^6 \text{ W}$$

Q.17 A circuit consists of two parallel resistors having resistances of 20 Ω and 30 Ω respectively, connected in series with a 15 Ω resistor. If the current through 30 Ω resistor is 1.2 A, find
i) Currents in 20 Ω and 15 Ω resistors.
ii) The voltage across the whole circuit.
iii) Voltage across 15 Ω resistor and 20 Ω resistor.
iv) Total power consumed in the circuit.

Ans. : The circuit is shown in the Fig. Q.17.1.

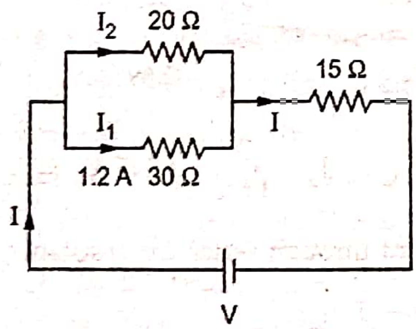

Fig. Q.17.1

i) The drop across parallel resistors is same

$\therefore \quad 1.2 \times 30 = 20 \times I_2$

$\therefore \quad I_2 = 1.8 \text{ A}$

$\therefore \quad I_{15\Omega} = I = I_1 + I_2 = 3 \text{ A}$

ii) V = Drop across 30 Ω + Drop across 15 Ω

$= 1.2 \times 30 + 3 \times 15 = 81 \text{ V}$

iii) Voltage across 15 Ω = 15 × 3 = 45 V

Voltage across 20 Ω = 20 × 1.8 = 36 V

iv) Power consumed in the circuit =

$I_1^2 \times 30 + I_2^2 \times 20 + I^2 \times 15$ or $= V \times I$

$= 43.2 + 64.8 + 135 = 81 \times 3 = 243 \text{ W}$

Q.18 Calculate the power absorbed by each component in the circuit shown in Fig. Q.18.1.

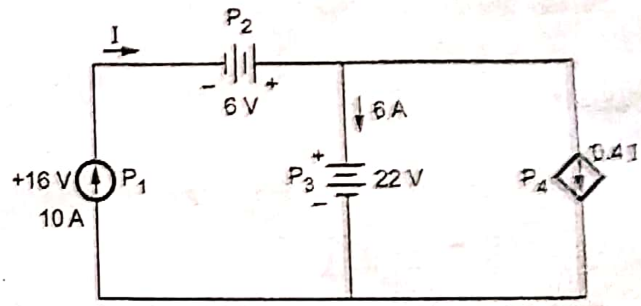

Fig. Q.18.1

Ans. : The power is given by V×I and I = 10 A.

According to sign convention for power,

$P_1 = (16)(-10) = -160 \text{ W}$ ... Supplied

$P_2 = (6)(-10) = -60 \text{ W}$ ... Supplied

$P_3 = (22)(+6) = +132 \text{ W}$ ... Absorbed

As 22 V and 0.4 I in parallel, the voltage across 0.4 I is 22 V and 0.4 I = 0.4×10 = 4 A.

$\therefore \quad P_4 = (22)(+4) = +88 \text{ W}$ ... Absorbed

$$\sum P = P_1 + P_2 + P_3 + P_4$$

$$= -160 - 60 + 132 + 88 = 0 \text{ W}$$

∴ Power supplied = Power absorbed

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1.8 : Current Division in Parallel Resistances

Q.19 Explain the curent division in two resistances connected in parallel.

[JNTU : Part A : Aug-08, May-13, Marks 3]

Ans. : • Consider a parallel circuit of two resistors R_1 and R_2 connected across a source of V volts.

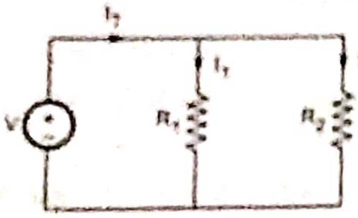

Fig. Q.19.1

• Current through R_1 is I_1 and R_2 is I_2, while total current drawn from source is I_T.

$$\therefore \quad I_T = I_1 + I_2$$

But $\quad I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}$

i.e. $\quad V = I_1 R_1 = I_2 R_2$

$$\therefore \quad I_1 = I_2\left(\frac{R_2}{R_1}\right)$$

Substituting value of I_1 in I_T,

$$\therefore \quad I_T = I_2\left(\frac{R_2}{R_1}\right) + I_2$$

$$\therefore \quad I_2 = \left[\frac{R_1}{R_1 + R_2}\right] I_T$$

Now $\quad I_1 = I_T - I_2$

$$= I_T - \left[\frac{R_1}{R_1 + R_2}\right] I_T$$

$$\therefore \quad I_1 = \left[\frac{R_2}{R_1 + R_2}\right] I_T$$

Key Point : *In general, the current in any branch is equal to the ratio of opposite branch resistance to the total resistance value, multiplied by the total current in the circuit.*

Q.20 Find the current in each branch of the below circuit shown in Fig. Q.20.1, using current division formula. [JNTU : Part B, Dec.-11, Marks 5]

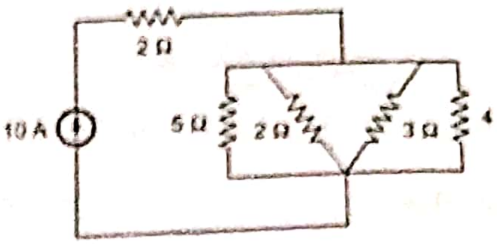

Fig. Q.20.1

Ans. : 2 Ω and 5 Ω are in parallel and 3 Ω and 4 Ω are in parallel.

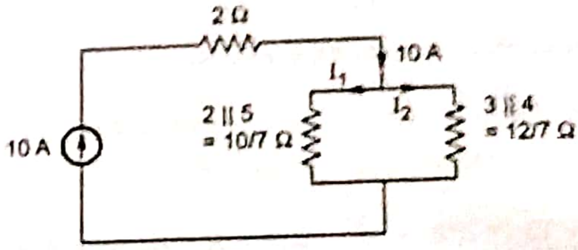

Fig. Q.20.1 (a)

Using current division rule,

$$I_1 = 10 \times \frac{(12/7)}{\left(\frac{10}{7} + \frac{12}{7}\right)} = 5.4545 \text{ A}$$

$$I_2 = 10 \times \frac{(10/7)}{\left(\frac{10}{7} + \frac{12}{7}\right)} = 4.5454 \text{ A}$$

Each I_1 and I_2 again flowing through parallel combination.

$$\therefore \quad I_{5\Omega} = I_1 \times \frac{2}{2+5} = 1.5584 \text{ A},$$

$$I_{2\Omega} = I_1 \times \frac{5}{2+5} = 3.8957 \text{ A}$$

$$\therefore \quad I_{3\Omega} = I_2 \times \frac{4}{4+3} = 2.5973 \text{ A},$$

$$I_{4\Omega} = I_2 \times \frac{3}{4+3} = 1.948 \text{ A}$$

While current through series 2 Ω resistance is 10 A.

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Q.21 Two resistances of 50 Ω and 40 Ω respectively are connected in parallel. A third resistance of 10 Ω is connected in series with the combination and a d.c. supply of 220 V is applied to the ends of the completed circuit. Calculate the current in each resistance.

[JNTU : Part B, Dec.-12, May-08, Marks 5]

Ans. : The circuit is shown in the Fig. Q.21.1 (a).

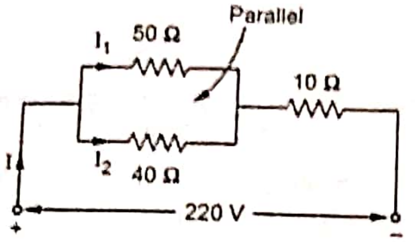

Fig. Q.21.1 (a)

$$50 \parallel 40 = \frac{50\times 40}{50+40} = 22.222\ \Omega$$

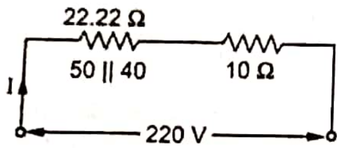

Fig. Q.21.1 (b)

From Fig. Q.21.1 (b),

$$I = \frac{V}{22.222+10} = \frac{220}{32.222} = 6.827\ \text{A}$$

Using current division rule for Fig. Q.21.1 (a),

$$I_1 = I\times\frac{40}{50+40} = 3.0344\ \text{A}$$

$$I_2 = I\times\frac{50}{50+40} = 3.7931\ \text{A}$$

1.9 : Star - Delta Transformation

Q.22 Define star and delta connection of resistances.

[JNTU : Part A, May-04, Aug.-06, Dec.-08, Marks 3]

Ans. : If the three resistances are connected in such a manner that one end of each is connected together to form a junction point called **Star point**, the resistances are said to be connected in **Star**.

The Fig. Q.22.1 (a), (b) and (c) show star connected resistances. The star point is indicated as S.

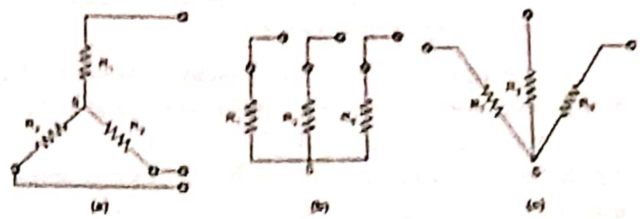

Fig. Q.22.1 Star connection of three resistances

- If the three resistances are connected in such a manner that one end of the first is connected to first end of second, the second end of second to first end of third and so on to complete a loop then the resistances are said to be connected in **Delta**.
- The Fig. Q.22.2 (a), (b) and (c) show delta connection of three resistances.

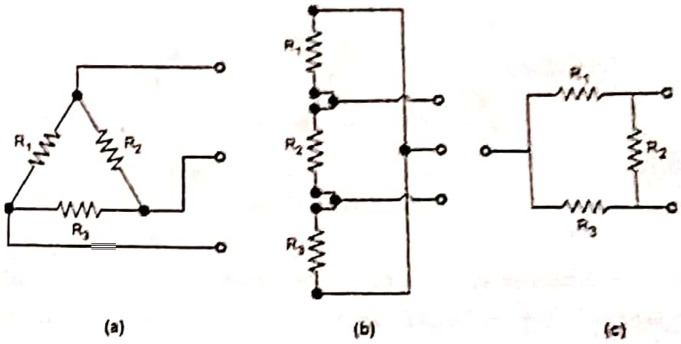

Fig. Q.22.2 Delta connection of three resistances

- Delta connection always forms a loop, closed path.

Q.23 Derive equations to convert delta connected resistances to equivalent star connected resistances.

[JNTU : Part B, May-05, 06, 09, Dec.-07, 11, 12, 16, Aug.-17, Marks 5]

Ans. : Consider the three resistances R_{12}, R_{23}, R_{31} connected in Delta as shown in the Fig. Q.23.1.

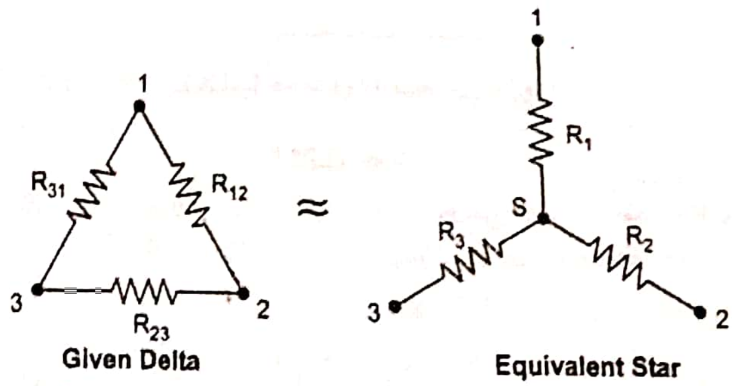

Fig. Q.23.1

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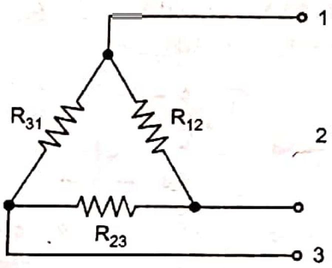
- The terminals between which these are connected in Delta are named as 1, 2 and 3.
- It is always possible to replace these Delta connected resistances by three equivalent Star connected resistances R_1, R_2, R_3 between the same terminals 1, 2, and 3. Such a Star is shown inside the Delta in the Fig. Q.23.1 which is called **equivalent Star of Delta connected resistances.**

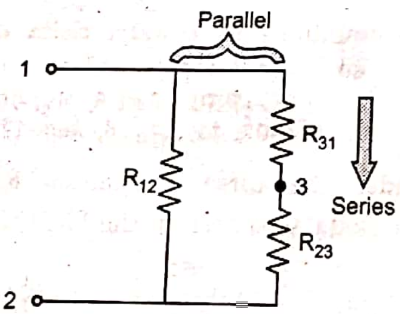
> To call these two arrangements as equivalent, the resistance between any two terminals must be same in both the types of connections.

- Let us analyse Delta connection first, shown in the Fig. Q.23.2 (a).



(a) Given Delta

- Find equivalent resistance between 1 and 2. We can redraw the network as viewed from the terminals (1) and (2), without considering terminal 3. This is shown in the Fig. Q.23.2 (b).



(b) Equivalent between 1 and 2

Fig. Q.23.2

- Between terminals (1) and (2) we get the combination as, R_{12} parallel with $(R_{31}+R_{23})$.

∴ Between (1) and (2) the resistance is,

$$= \frac{R_{12}(R_{31}+R_{23})}{R_{12}+(R_{31}+R_{23})} \quad ...(1)$$

- Consider the same two terminals of equivalent Star connection shown in the Fig. Q.23.3.

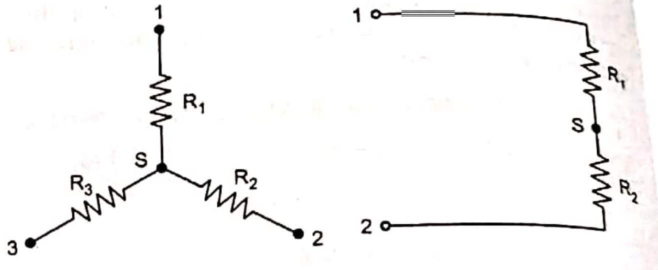


Fig. Q.23.3

- Terminal (3) is not getting connected anywhere and hence is not playing any role.

∴ Between terminals (1) and (2) the resistance is

$$= R_1 + R_2 \quad ...(2)$$

Equating equations (1) and (2),

$$\frac{R_{12}(R_{31}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_1 + R_2 \quad ...(3)$$

- Similarly if we find the equivalent resistance as viewed through terminals (2) and (3) in both the cases and equating, we get,

$$\frac{R_{23}(R_{31}+R_{12})}{R_{12}+(R_{23}+R_{31})} = R_2 + R_3 \quad ...(4)$$

- Simiarly if we find the equivalent resistance as viewed through terminals (3) and (1) in both the cases and equating, we get,

$$\frac{R_{31}(R_{12}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_3 + R_1 \quad ...(5)$$

- Subtracting equations (4) from (3),

$$\frac{R_{12}(R_{31}+R_{23})-R_{23}(R_{31}+R_{12})}{(R_{12}+R_{23}+R_{31})} = R_1 + R_2 - R_2 - R_3$$

$$\therefore \quad R_1 - R_3 = \frac{R_{12}\,R_{31}-R_{23}\,R_{31}}{R_{12}+R_{23}+R_{31}} \quad ...(6$$

- Adding equations (6) and (5),

$$\frac{R_{12}\,R_{31}-R_{23}\,R_{31}+R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} = R_1 + R_3 + R_1 - R$$

$$\therefore \quad \frac{R_{12}\,R_{31}-R_{23}\,R_{31}+R_{31}\,R_{12}+R_{31}\,R_{23}}{R_{12}+R_{23}+R_{31}} = 2R$$

i.e.
$$2R_1 = \frac{2 R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

Similarly by using another combinations of subtraction and addition with equations (3), (4) and (5) we can get,

$$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}} \quad \text{and} \quad R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$$

Important Point to Remember

- The equivalent star resistance between any terminal and star point is equal to the product of the two resistances in delta, which are connected to same terminal, divided by the sum of all three delta connected resistances.

Q.24 Derive equations to convert star connected resistances to equivalent delta connected resistances.

[JNTU : Part B, May-05, 06, 09, Dec.-07, 11, 12, Aug.-17, Marks 5]

Ans. : Consider the three resistances R_1, R_2 and R_3 connected in Star as shown in Fig. Q.24.1.

- By Star-Delta conversion, it is always possible to replace these Star connected resistances by three equivalent Delta connected resistances R_{12}, R_{23} and R_{31}, between the same terminals. This is called **equivalent Delta of the given star.**
- We are interested in finding out values of R_{12}, R_{23} and R_{31} interms of R_1, R_2 and R_3.
- From the result of Delta-Star transformation we know that,

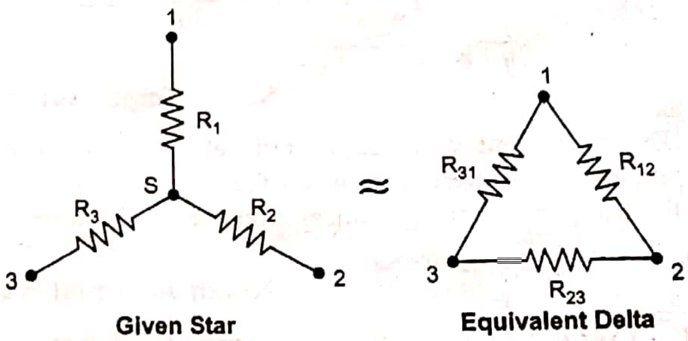


Fig. Q.24.1

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} \quad \text{...(1)}$$

$$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}} \quad \text{...(2)}$$

$$R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \quad \text{...(3)}$$

- Now multiply (1) and (2), (2) and (3), (3) and (1) to get following three equations.

$$R_1 R_2 = \frac{R_{12}^2 R_{31} R_{23}}{(R_{12} + R_{23} + R_{31})^2} \quad \text{...(4)}$$

$$\therefore \quad R_2 R_3 = \frac{R_{23}^2 R_{12} R_{31}}{(R_{12} + R_{23} + R_{31})^2} \quad \text{...(5)}$$

$$R_3 R_1 = \frac{R_{31}^2 R_{12} R_{23}}{(R_{12} + R_{23} + R_{31})^2} \quad \text{...(6)}$$

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- Now add (4), (5) and (6)

$$\therefore \quad R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}{}^2R_{31}R_{23} + R_{23}{}^2R_{12}R_{31} + R_{31}{}^2R_{12}R_{23}}{(R_{12}+R_{23}+R_{31})^2}$$

$$\therefore \quad R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{31}R_{23}(R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2} = \frac{R_{12}R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}$$

But $$\frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}} = R_1 \quad \text{... From equation (1)}$$

$\therefore$ Substituting in above in R.H.S. we get,

$$\therefore \quad R_1R_2 + R_2R_3 + R_3R_1 = R_1 R_{23}$$

$$\therefore \quad \boxed{R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}}$$

- Similarly substituting in R.H.S., remaining values from equations (2) and (3), we can write relations for remaining two resistances as,

$$\boxed{R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3} \quad \text{and} \quad R_{31} = R_3 + R_1 + \frac{R_1 R_3}{R_2}}$$

Important Points to Remember

- The equivalent delta connected resistance to be connected between any two terminals is sum of the two resistances connected between the same two terminals and star point respectively in star, plus the product of the same two star resistances divided by the third star resistance.

Result for equal resistances in star and delta

- If all resistances in a Delta connection have same magnitude say R, then its equivalent Star will contain,

$$R_1 = R_2 = R_3 = \frac{R \times R}{R+R+R} = \frac{R}{3}$$

- If all three resistances in a Star connection are of same magnitude say R, then its equivalent Delta contains all resistances of same magnitude of,

$$R_{12} = R_{31} = R_{23} = R + R + \frac{R \times R}{R} = 3\,R$$

Q.25 By using star-delta transformation for the following Fig. Q.25.1, find the current 'I' supplied by the battery ?

[JNTU : Part B, Dec.-16, Marks 5]

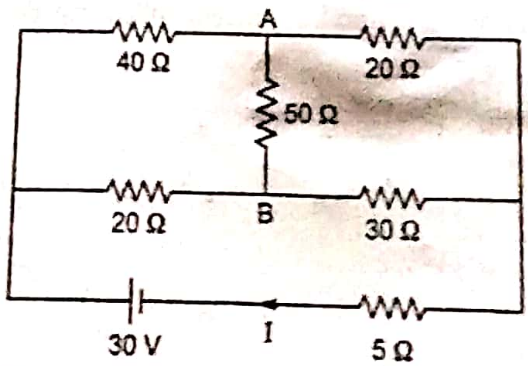

Fig. Q.25.1

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Ans. : Convert the ΔABC to equivalent star.

$$R_1 = \frac{20\times 50}{20+50+30} = 10\Omega$$

$$R_2 = \frac{30\times 50}{20+50+30} = 15\Omega$$

$$R_3 = \frac{20\times 30}{20+50+30} = 6\Omega$$

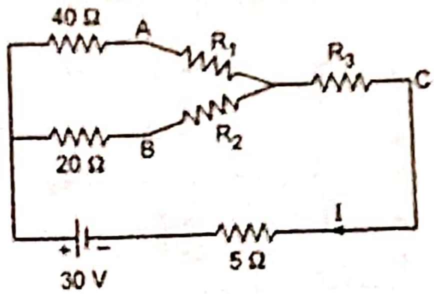

Fig. Q.25.1 (a)

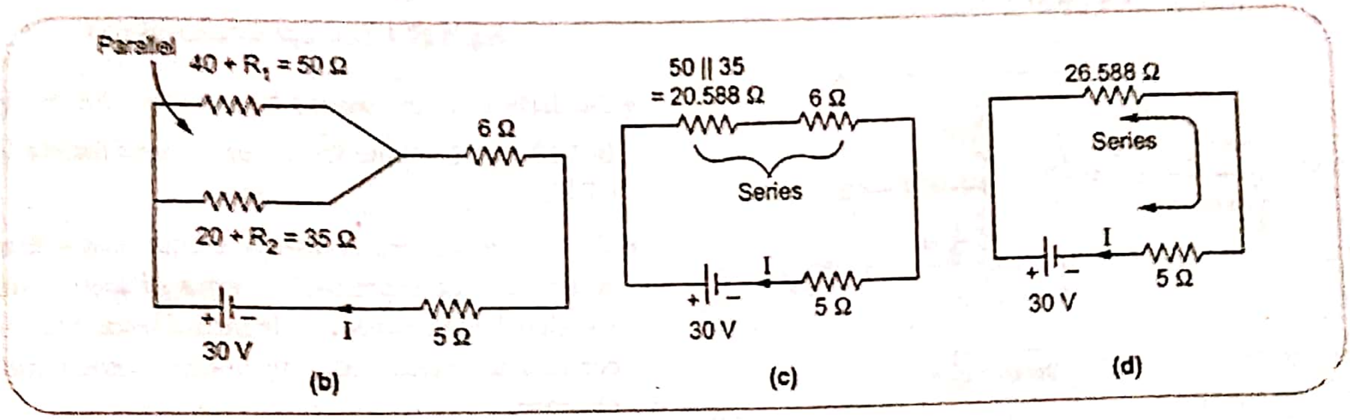

Fig. Q.25.1

$$\therefore \quad I = \frac{30}{5+26.588} = 0.9497A$$

Q.26 Making use of star/delta transformation, determine the resistance between terminals A and B shown in Fig. Q.26.1. [JNTU : May-18, Marks 5]

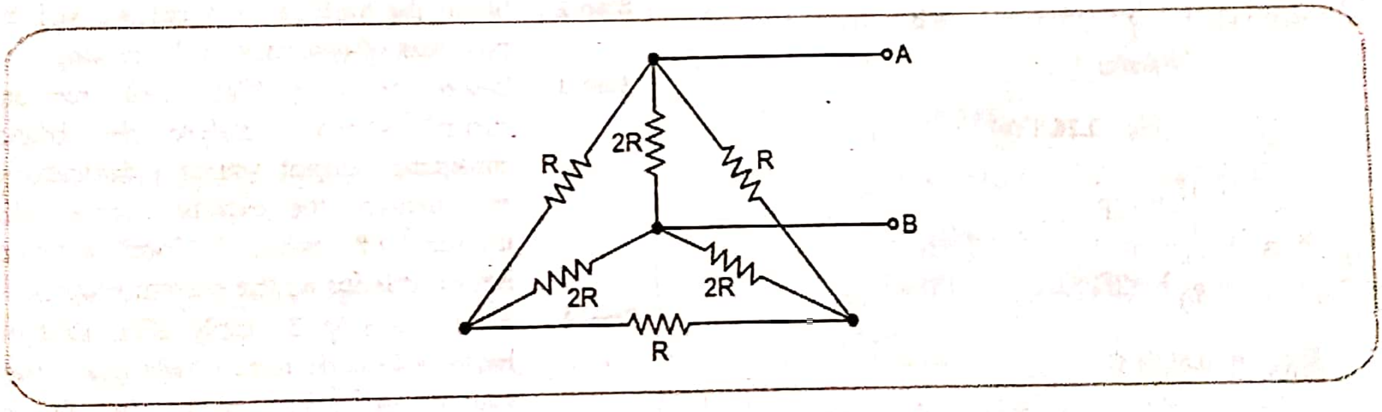

Fig. Q.26.1

Ans. : Convert lower delta to star

$$R_1 = \frac{2R\times 2R}{2R+2R+R} = \frac{4}{5}R$$

$$R_2 = R_3 = \frac{R\times 2R}{2R+2R+R} = \frac{2}{5}R$$

The circuit becomes as shown in the Fig. Q.26.1 (b)

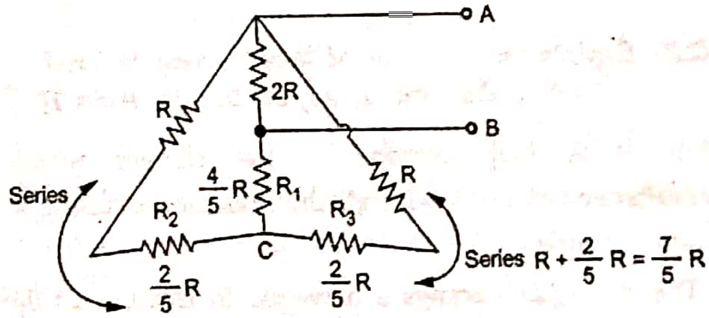

Fig. Q.26.1 (b)

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Redrawing the circuit as shown in the Fig. Q. 26.1 (c).

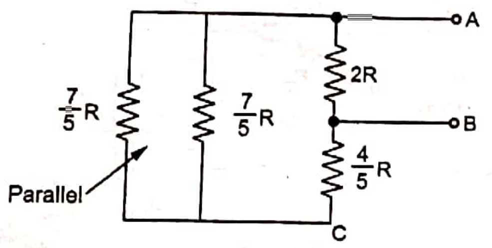

Fig. Q.26.1 (c)

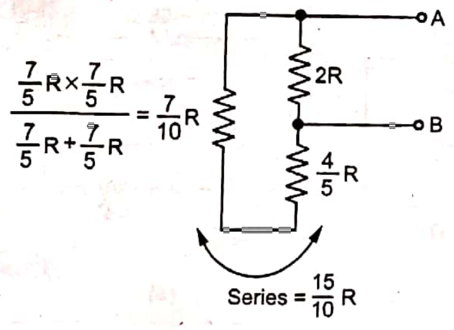

Fig. Q.26.1 (d)

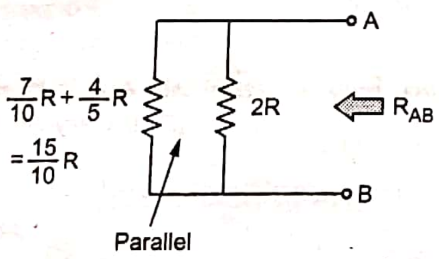

Fig. Q.26.1 (e)

$$\therefore \quad R_{AB} = \frac{\frac{15}{10}R\times 2R}{\frac{15}{10}R+2R}$$

$$\therefore \quad R_{AB} = \mathbf{0.8571\ R}$$

1.10 : Loop Analysis (Mesh Analysis)

Q.27 Explain the concept of loop current in brief.
[JNTU : Part A, May-06, Dec.-14, Marks 3]

Ans. : • A loop current is that current which simultaneously links with all the branches, defining a particular loop.

- The Fig. Q.27.1 shows a network. In this circuit, I_1 is the loop current for the loop ABFEA and simultaneously links with the branches AB, BF, FE and EA.

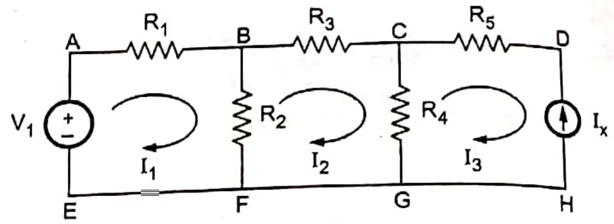

Fig. 1.27.1 Concept of loop current

- Similarly I_2 is the second loop current for the loop BCGFB and I_3 is the third loop current for the loop CDFGC.
- The branch current is always unique hence branch current can be expressed in terms of loop currents associated with particular branch. Hence once loop currents are calculated, any branch current can be obtained.
- Note that the branches consisting current sources, directly decide the value of the branch currents flowing through them.

Steps for the application of loop analysis

Step 1 : Choose the various loops.

Step 2 : Show the various loop curents and the polarities of associated voltage drops.

Step 3 : Before applying KVL, look for any current source. Analyse the branch consisting current source independently and express the current source value interms of assumed loop currents. Repeat this for all the current sources.

Step 4 : After the step 3, apply KVL to those loops, which do not include any current source. A loop cannot be defined through current source from KVL point of view. Follow the sign convention.

Step 5 : Solve the equations obtained in step 3 and step 4 simultaneously, to obtain required unknowns.

Q.28 Calculate V_0 and I_0 for the circuit shown in Fig. Q.28.1. **[JNTU : Part B, May-15, Marks 5]**

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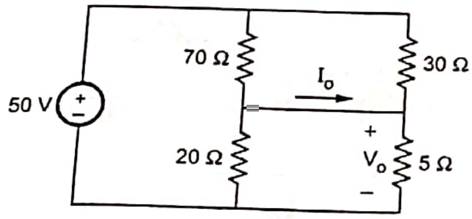

Fig. Q.28.1

Ans. : Use loop analysis. The loop currents and polarities of various drops are shown in the Fig. Q.28.1(a). Apply KVL to the three loops,

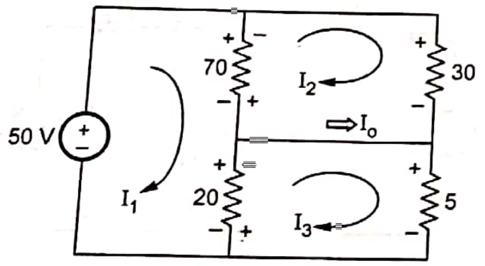

Fig. Q.28.1(a)

$$-70I_1 + 70I_2 - 20I_1 + 20I_3 + 50 = 0$$

i.e. $$-90I_1 + 70I_2 + 20I_3 = -50$$

$$-30I_1 - 70I_2 + 70I_1 = 0$$

i.e. $$70I_1 - 100I_2 = 0$$

$$-5I_3 - 20I_3 + 20I_1 = 0$$

i.e. $$20I_1 - 25I_3 = 0$$

Solving, $I_1 = 2$ A, $I_2 = 1.4$ A, $I_3 = 1.6$ A

$\therefore$ $I_0 = I_3 - I_2 = 1.6 - 1.4 = \mathbf{0.2\ A} \rightarrow$

$V_0 = 5I_3 = 5 \times 1.6 = \mathbf{8\ V}$

Q.29 Determine the voltage drop across the 10 Ω resistance for the following Fig. Q.29.1.

☞[JNTU : Part B, May-11, Marks 5]

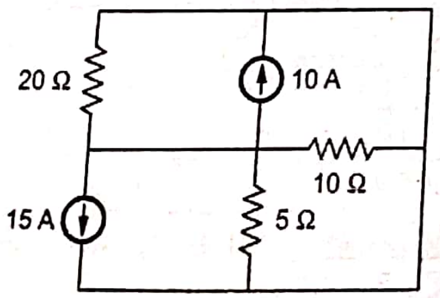

Fig. Q.29.1

Ans. : Assuming loop currents as shown in the Fig. Q.29.1 (a).

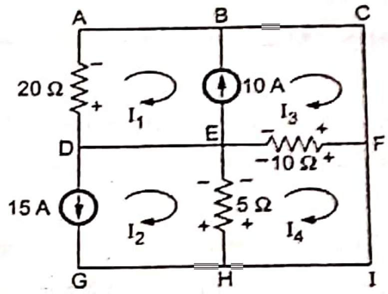

Fig. Q.29.1 (a)

Analyse the branches containing only current sources first.

From branch BE, $-I_1 + I_3 = 10$ ...(1)

From branch DG, $I_2 = -15$ A ...(2)

Applying KVL to the loops,

Loop ABCFEDA,

$$-10I_3 + 10I_4 - 20I_1 = 0$$

i.e. $20I_1 + 10I_3 - 10I_4 = 0$ ...(3)

Loop EFIHE, $-10I_4 + 10I_3 - 5I_4 + 5I_2 = 0$ and use equation (2),

$\therefore 10I_3 - 15I_4 = 75$ ...(4)

Solving equations (1), (3) and (4) simultaneously,

$$I_1 = -3.5714\text{ A},\ I_3 = 6.4286\text{ A},$$

$$I_4 = -0.7143\text{ A}$$

$\therefore$ $I_{10\Omega} = I_3 - I_4 = 6.4286 - (-0.7143)$

$= 7.1429$ A (F to E)

$\therefore$ $V_{10\Omega} = 10 \times I_{10\Omega} = \mathbf{71.429\ V\ with\ F\ positive}$

1.11 : Node Analysis

Q.30 Explain in detail about node analysis.

☞ [JNTU : Part A, May-06,17, Dec.-14, Marks 5]

Ans. : Steps for the application of node analysis

Step 1 : Choose the base node, major nodes and node voltages to be obtained. The junction point where three or more branches meet are the major nodes.

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UNIT - I

2 A.C. Circuits

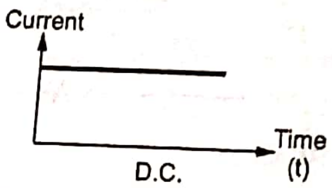
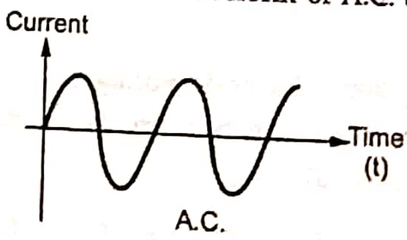
2.1 : Introduction to Alternating Waveform

Q.1 Define alternating quantity. [JNTU : Part A, May-15,17, Marks 2]

Ans. :
- An alternating current (a.c.) is the current which changes periodically both in magnitude and direction.
- In alternating waveform there are two half cycles, one positive and other negative. These two half cycles make one cycle. Current increases in magnitude, in one particular direction, attains maximum and starts decreasing, passing through zero it increases in opposite direction and behaves similarly.

Q.2 Give comparison between a.c. and d.c. [JNTU : Part A, May-03, 07, Dec.-09, Marks 3]

Ans. :

Sr. No.	D.C.	A.C.
1.	D.C. is direct current having constant magnitude and direction.	A.C. is alternating current whose direction and magnitude changes with time.
2.	The frequency of D.C. is zero.	There is finite frequency of A.C. It is 50 Hz in our nation.
3.	Raising and lowering of voltages is not easy.	Raising and lowering of voltages is very easy with the help of transformers.
4.	It is difficult to obtain A.C. from D.C.	It is easy to obtain D.C. from A.C. using rectifiers.
5.	The D.C. machines are expensive and require frequent maintenance.	The A.C. machines are cheaper and require less maintenance.
6.	The Figure shows the waveform of D.C. current. 	The Figure shows the waveform of A.C. current. 

Q.3 : Mention the advantages of sinusoidal alternating quantity. [JNTU : Part A, May-17, Marks 3]

Ans. :

1. Mathematically very easy to write the equations.
2. Any other complex waveform can be resolved into a series of sine and cosine waveforms.
3. The sinusoidal waveform is the only waveform which passes through linear elements like R, L and C without any distortion.

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4. The analysis of electrical circuits is easy because derivative and integration of sinusoidal function is again a sinusoidal function.

2.2 : Representation of Sinusoidal Waveforms

Q.4 Sketch the sinusoidal alternating waveform and define : i) Instantaneous value ii) Waveform iii) Cycle iv) Frequency v) Time period vi) Amplitude.

[JNTU : Part B, Dec.-16, Marks 5]

Ans. : The Fig. Q.4.1 shows the graphical representation of an alternating quantity.

i) **Instantaneous value :** The value of an alternating quantity at a particular instant is known as its instantaneous value. e.g. e_1 and $-e_2$ are the instantaneous values of an alternating e.m.f. at the instants t_1 and t_2 respectively shown in the Fig. Q.4.1 .

ii) **Waveform :** The graph of instantaneous values of an alternating quantity plotted against time is called its **waveform.**

iii) **Cycle :** Each repetition of a set of positive and negative instantaneous values of the alternating quantity is called a **cycle.**

- A **cycle** can also be defined as that interval of time during which a complete set of non-repeating events or waveform variations occur (containing positive as well as negative loops).
- One cycle corresponds to 2π radians or 360°.

iv) **Time period (T) :** The time taken by an alternating quantity to complete its one cycle is known as its **time period** denoted by **T seconds.**

- After every T seconds, the cycle of an alternating quantity repeats.

v) **Frequency (f) :** The number of cycles completed by an alternating quantity per second is known as its **frequency.** It is denoted by f and it is measured in **cycles / second** which is known as **Hertz,** denoted as **Hz.**

- Frequency is reciprocal of the time period.

$$f = \frac{1}{T} \text{ Hz}$$

- **In our nation, standard frequency of alternating voltages and currents is 50 Hz.**

vi) **Amplitude :** The maximum value attained by an alternating quantity during positive or negative half cycle is called its **amplitude.** It is denoted as E_m or I_m.

- Thus E_m is called peak value of the voltage while I_m is called peak value of the current.
- The amplitude is also called peak value or maximum value of an alternating quantity.

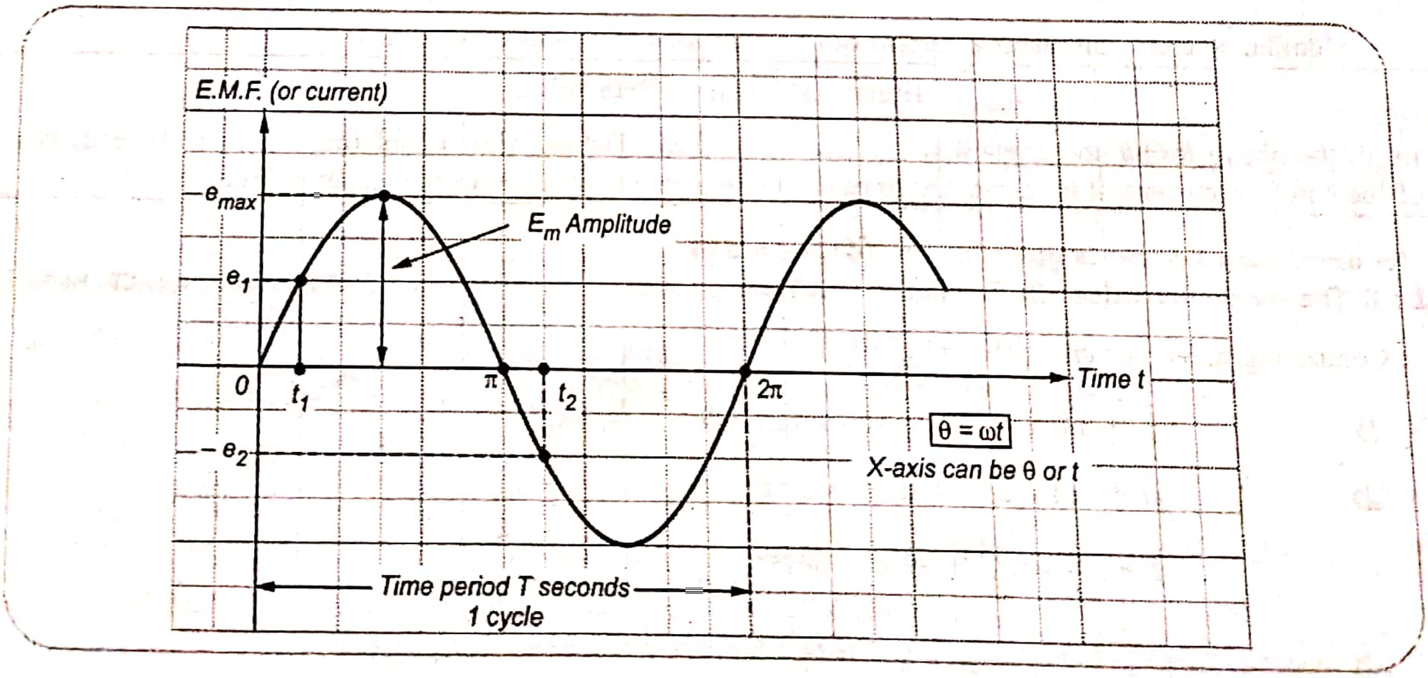

Fig. Q.4.1 Graphical representation of an alternating quantity

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Important Points to Remember

- The angular frequency ω is related with frequency f as $\omega = 2\pi f$ and measured in radians per second. As $f = \frac{1}{T}$, $\omega = \frac{2\pi}{T}$ rad/sec.
- The angle θ and ω are related with each other through time t as $\theta = \omega t$ rad. Hence θ is always measured in radians.

2.3 : Equation of an Alternating Quantity

Q.5 Explain the different representations of sinusoidal quantities. [JNTU : Part B, Dec.-14, May-17, Marks 5]

Ans. : • As the standard waveform of an alternating quantity is purely sinusoidal, the equation of an alternating voltage can be expressed as,

$$e(t) = E_m \sin\theta \text{ volts}$$

where, E_m = Amplitude or maximum or peak value of the voltage.

e = Instantaneous value of an alternating voltage.

• Similarly equation of an alternating current can be expressed as,

$$i(t) = I_m \sin\theta$$

where, I_m = Amplitude or maximum or peak value of the current.

i = Instantaneous value of an alternating current

• **The equation can be expressed in various forms as :**

As, $\theta = \omega t$ radians, $\omega = 2\pi f$ rad / sec. and $f = \frac{1}{T}$

$$\therefore \quad e = E_m \sin(\omega t) = E_m \sin(2\pi f t) = E_m \sin\left(\frac{2\pi}{T}t\right)$$

• Similar to e.m.f., alternating current can be expressed in all the above forms.

Important Point to Remember

- In all the above forms, the angle θ is expressed in radians. Hence, while calculating the instantaneous value of the e.m.f. or current, it is necessary to calculate the sine of the angle expressed in radians.

Q.6 An alternating current is given by $i = 141.4 \sin 314 t$.
Find : i) The maximum value ii) Frequency iii) Time period. [JNTU : Part A, May-09, Marks 3]

Ans. : Comparing given current with $i = I_m \sin\omega t$,

i) $I_m = 141.4$ A ... **Maximum value**

ii) $\omega = 314$ rad/s but $\omega = 2\pi f$

$$\therefore \quad f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = 49.97 \approx 50 \text{ Hz}$$

iii) $$T = \frac{1}{f} = \frac{1}{50} = 20 \text{ ms}$$

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2.4 : R.M.S. or Effective Value of an Alternating Quantity

Q.7 Define R.M.S. or effective value of an alternating quantity.

[JNTU : Part A, May-05, 09, 12, 15, 17, Dec.-08, 12, Marks 2]

Ans. : The effective or r.m.s. value of an alternating current is given by that steady current (D.C.) which, when flowing through a given circuit for a given time, produces the same amount of heat as produced by the alternating current, which when flowing through the same circuit for the same time.

Q.8 Derive the relation between r.m.s. value and the maximum value of an alternating quantity.

[JNTU : Part B, May-08, 18, Dec.-06, 14, 18, Marks 5]

Ans. : Consider sinusoidally varying alternating current and square of this current as shown in the Fig. Q.8.1.

Step 1 : The current, $i = I_m \sin\theta$

Step 2 : Square of current, $i^2 = I_m^2 \sin^2\theta$

- The area of curve over half a cycle can be calculated by considering an interval dθ as shown.

Area of square curve over half cycle $= \int_0^{\pi} i^2 \, d\theta$ and length of the base is π.

Step 3 :

∴ Average value of square of the current over half cycle is,

$$= \frac{\text{Area of curve over half cycle}}{\text{Length of base over half cycle}} = \frac{\int_0^{\pi} i^2 \, d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} i^2 \, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi}\left[\frac{1-\cos 2\theta}{2}\right] d\theta = \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}[\pi] = \frac{I_m^2}{2}$$

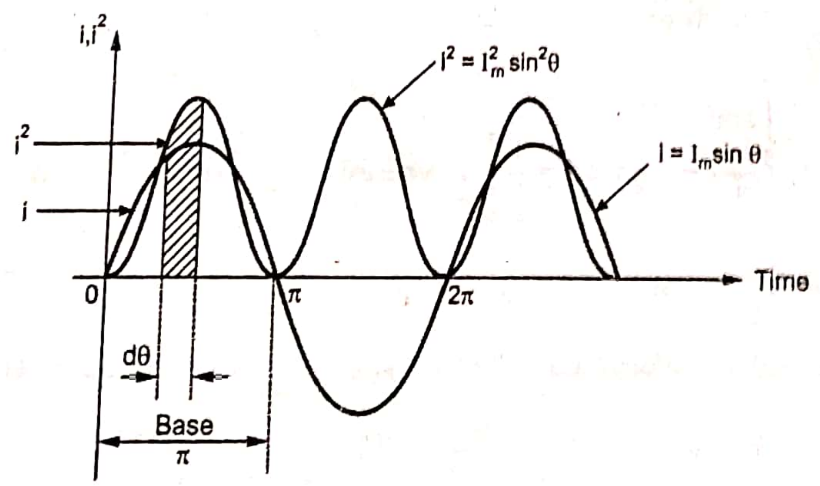

Fig. Q.8.1 Waveform of current and square of the current

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Step 4 : Root mean square value i.e. r.m.s. value can be calculated as,

$$I_{r.m.s.} = \sqrt{\text{Mean or average of square of current}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$\therefore \quad I_{r.m.s.} = \frac{I_m}{\sqrt{2}} = 0.707\ I_m \quad \text{and} \quad V_{r.m.s.} = \frac{V_m}{\sqrt{2}} = 0.707\ V_m$$

- **The r.m.s. value of the sinusoidal alternating current is 0.707 times the maximum or peak value or amplitude of that alternating current.**
- **The above result is also applicable to sinusoidal alternating voltages.**

2.5 : Average Value of an Alternating Quantity

Q.9 Define average value of an alternating quantity. [JNTU : Part A, May-05, 09, 12, 15, Dec.-08, 12, Marks 2]

Ans. : The average value of an alternating quantity is defined as that value which is obtained by averaging all the instantaneous values over a period of half cycle.

- **For a symmetrical a.c., the average value over a complete cycle is zero** as both positive and negative half cycles are exactly identical. Hence, the average value is defined for half cycle only.

Q.10 Obtain the relation between average value and the maximum value of an alternating quantity. [JNTU : Part B, May-08,18, Dec.-09,, Marks 5]

Ans. : Consider sinusoidally varying current,

$I = I_m \sin \theta$

- Consider the elementary interval of instant '$d\theta$' as shown in the Fig. Q.10.1.
- The average instantaneous value of current in this interval is say, 'i' as shown.
- The average value can be obtained by taking ratio of area under curve over **half cycle** to length of the base for half cycle.

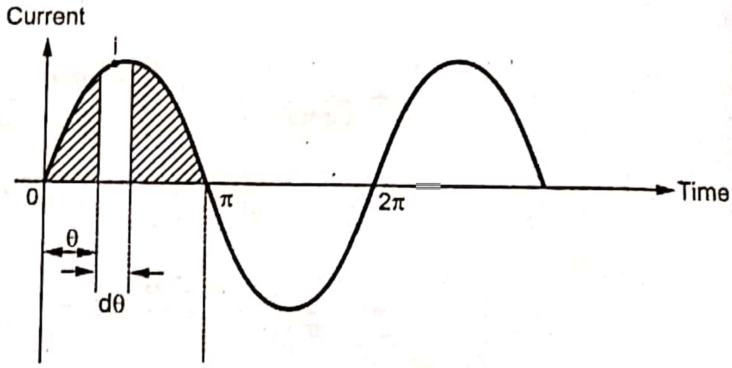

Fig. Q.10.1 Average value of an alternating current

$$\therefore \quad I_{av} = \frac{\text{Area under curve for half cycle}}{\text{Length of base over half cycle}}$$

$$I_{av} = \frac{\int_0^{\pi} i\, d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} i\ d\theta = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta = \frac{I_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{\pi}[-\cos\pi + \cos 0] = \frac{I_m}{\pi}[2] = \frac{2 I_m}{\pi} = 0.637\ I_m$$

- For a purely sinusoidal waveform, the average value is expressed in terms of its maximum value as,

$$\therefore \quad I_{av} = 0.637\ I_m \quad \text{and} \quad V_{av} = 0.637\ V_m$$

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2.6 : Form Factor (K_f)

Q.11 Define form factor. State its value for purely sinusoidal quantity.

[JNTU : Part A, May-11, 15, 17, 18, Dec.-05, 16, Marks 2]

Ans. : The form factor of an alternating quantity is defined as the ratio of r.m.s. value to the average value,

Form factor, $K_f = \dfrac{\text{r.m.s. value}}{\text{average value}}$

- The form factor for sinusoidal alternating currents or voltages can be obtained as,

$$K_f = \frac{0.707\ I_m}{0.637\ I_m} = 1.11 \quad \text{for sinusoidally varying quantity}$$

2.7 : Peak or Crest Factor (K_p)

Q.12 Define peak factor. State its value for purely sinusoidal quantity.

[JNTU : Part A, May-11, 18, Dec.-05, 16, Marks 2]

Ans. : The peak factor of an alternating quantity is defined as ratio of maximum value to the r.m.s. value.

Peak factor $K_p = \dfrac{\text{maximum value}}{\text{r.m.s. value}}$

- The peak factor for sinusoidally varying alternating currents and voltages can be obtained as,

$$K_p = \frac{I_m}{0.707\ I_m} = 1.414 \quad \text{for sinusoidal waveform}$$

Q.13 Find the form factor of a square wave.

[JNTU : Part B, May-11, Dec.-15, Marks 5]

Ans. : A square wave is shown in the Fig. Q.13.1.

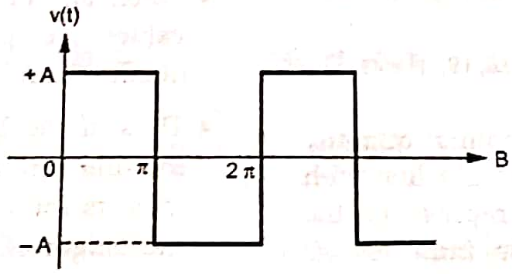

Fig. Q.13.1

$$V_{av} = \frac{\int_0^{\pi} v(t)\, d\theta}{\pi} = \frac{\int_0^{\pi} A\, d\theta}{\pi} = \frac{A[\theta]_0^{\pi}}{\pi} = \frac{A\pi}{\pi} = A$$

$$V_{rms} = \sqrt{\frac{\int_0^{\pi} v^2(t)\, d\theta}{\pi}} = \sqrt{\frac{\int_0^{\pi} A^2 d\theta}{\pi}} = \sqrt{\frac{A^2[\theta]_0^{\pi}}{\pi}} = \sqrt{\frac{A^2 \times \pi}{\pi}} = A$$

$$\therefore \quad K_f = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{A}{A} = 1$$

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Q.14 A wire carries a current, which is a combination of a. d.c. current of 10 A and a sinusoidal current with a peak value of 10 A. Determine RMS value of the resultant.

[JNTU : Part B, May-19, Marks 5]

Ans. : The resultant current is given by,

$$i_R = I_{dc} + I_m \sin\theta = 10 + 10 \sin\theta$$

$$\therefore \quad i_R(\text{r.m.s.}) = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_R^2 \, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (10+10\sin\theta)^2 \, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [100+200\sin\theta+100\sin^2\theta] \, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \left[100+200\sin\theta+100\left(\frac{1-\cos 2\theta}{2}\right)\right] d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\left[100\,\theta-200\cos\theta+\frac{100}{2}\theta-\frac{100\sin 2\theta}{2\times 2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{1}{2\pi}[200\,\pi-200+200+100\pi-0]} \qquad \ldots\cos 2\pi = +1$$

$$= \sqrt{\frac{1}{2\pi}\times 300\,\pi} = \mathbf{12.2475\ A}$$

2.8 : Phasor Representation

Q.15 What is phasor ? How a rotating phasor represents an alternating quantity ?

[JNTU : Part B, May-18,19, Marks 5]

Ans. : The sinusoidally varying alternating quantity can be represented graphically by a straight line with an arrow. The length of the line represents the magnitude of the quantity and arrow indicates its direction. Such a line is called a **phasor.**

- The phasors are assumed to be rotated in anticlockwise direction with a constant speed ω rad/sec.
- Consider a phasor 'Oa', rotating in anticlockwise direction, with uniform angular velocity, with its starting position 'a' as shown in the Fig. Q.15.1.
- If the projections of this phasor on Y-axis are plotted against the angle turned through 'θ', (or time as $\theta = \omega t$), we get a sine waveform.
- Consider the various positions shown in the Fig. Q.15.1.

1. At point 'a', the Y-axis projection is zero. The instantaneous value of the current is also zero.
2. At point 'b', the Y-axis projection is [l (Ob) sin θ)]. The length of the phasor is equal to the maximum value of an alternating quantity. So, instantaneous value of the current at this position is $i = I_m \sin\theta$, represented in the waveform.
3. At point 'c', the Y-axis projection 'Oc' represents entire length of the phasor i.e. instantaneous value equal to the maximum value of current I_m.
4. Similarly, at point d, the Y-axis projection becomes $I_m \sin\theta$ which is the instantaneous value of the current at that instant.

- Similarly, at points e, f, g, h the Y-axis projections give us instantaneous values of the current at the respective instants.
- When all such Y-axis projections i.e. instantaneous values are plotted, full cycle of the alternating quantity can be obtained.
- Thus, if the length of the phasor is taken equal to the maximum value of the alternating quantity, then its rotation in space at any instant is such that the length of its projection on the Y-axis gives the instantaneous value of the alternating quantity at that particular instant.

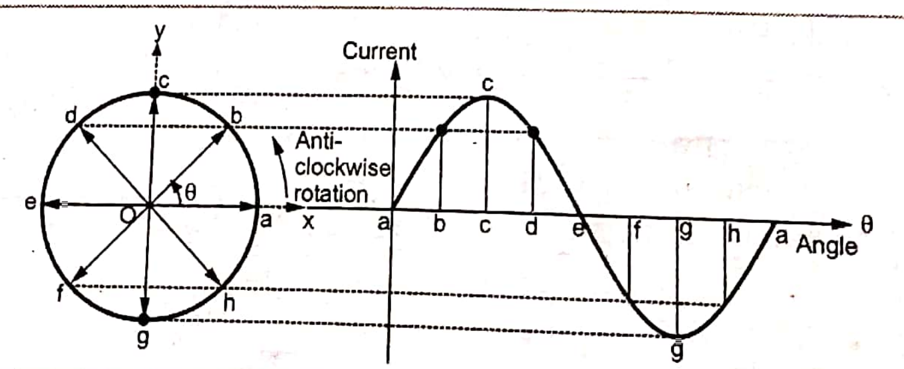

Fig. Q.15.1 Phasor representation of an alternating quantity

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Important Points to Remember

- In practice, the alternating quantities are represented by their r.m.s. values. Hence, the length of the phasor represents r.m.s. value of the alternating quantity.
- Phasors are always assumed to be rotated in **anticlockwise** direction.

2.9 : Concept of Phase and Phase Difference

Q.16 Explain the concept of phase and phase difference in alternating quantities.

[JNTU : Part A, Dec.-11, 18, Aug.-17, Marks 3]

Ans. : The phase of an alternating quantity at any instant is the angle ϕ (in radians or degrees) traveled by the phasor representing that alternating quantity upto the instant of consideration, measured from the reference.

- Let X-axis be the reference axis. So, phase of the alternating current shown in the Fig. Q.16.1 at the instant A is $\phi = 0°$.

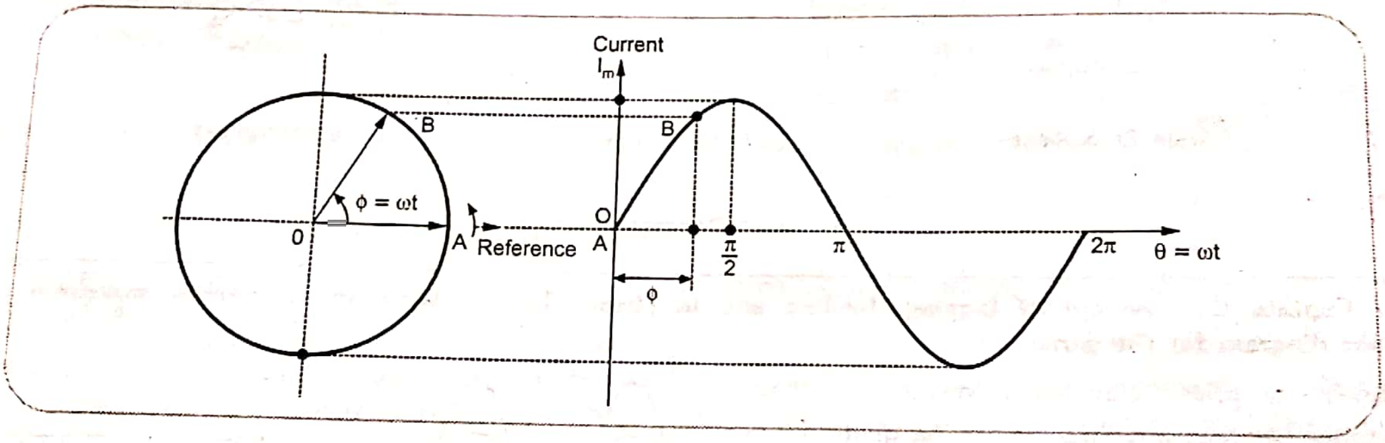

Fig. Q.16.1 Concept of phase

- While the phase of the current at the instant B is the angle ϕ through which the phasor has traveled, measured from the reference axis i.e. X-axis.
- In general, the phase ϕ of an alternating quantity varies from $\phi = 0$ to 2π radians or $\phi = 0°$ to $360°$.
- In the a.c. analysis, it is not necessary that all the alternating quantities must be always in phase. It is possible that if one is achieving its zero value and at the same instant the other is having some negative value or positive value then such two quantities are said to have **phase difference** between them.
- The difference between the phases of the two alternating quantities is called the **phase difference** which is nothing but the angle difference between the two phasors representing the two alternating quantities at a particular instant.

Important Point to Remember

- In terms of phase, the equation of an alternating quantity can be modified as,

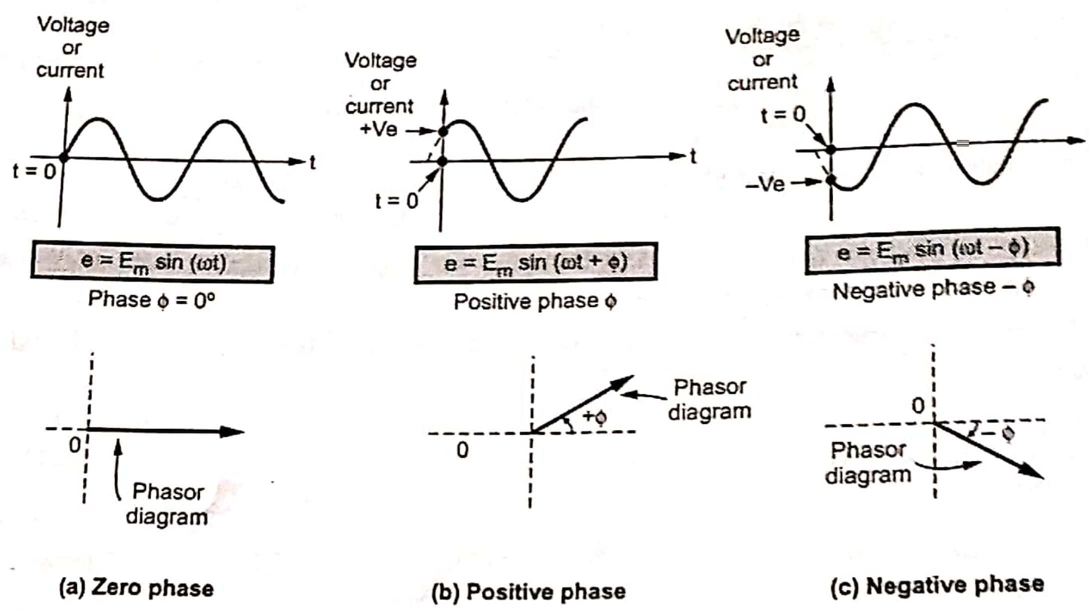

Fig. 2.9.1 Concept of phase

Q.17 Explain the concept of lagging, leading and in phase phasors. Draw the respective waveform and phasor diagram for the same.

Ans. : i) In phase Phasors : When the phase difference between any two alternating quantities is zero, they are said to be in phase. They reach maximum positive, negative and zero values at the same time though their amplitudes may be different. This is shown in the Fig. Q.17.1.

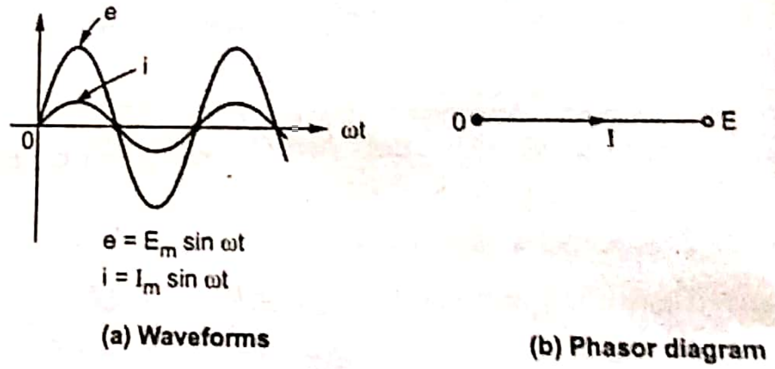

Fig. Q.17.1

ii) Lagging Phasor : If one alternating quantity achieves its positive, negative and zero values later compared to second alternating quantity, then it is said to be lagging with respect to second quantity. This is shown in the Fig. Q.17.2.

iii) Leading Phasor : If one alternating quantity achieves its positive, negative and zero values earlier compared to second alternating quantity, then it is said to be leading with respect to second quantity. This is shown in the Fig. Q.17.3.

(See Fig. Q.17.3 on next page).

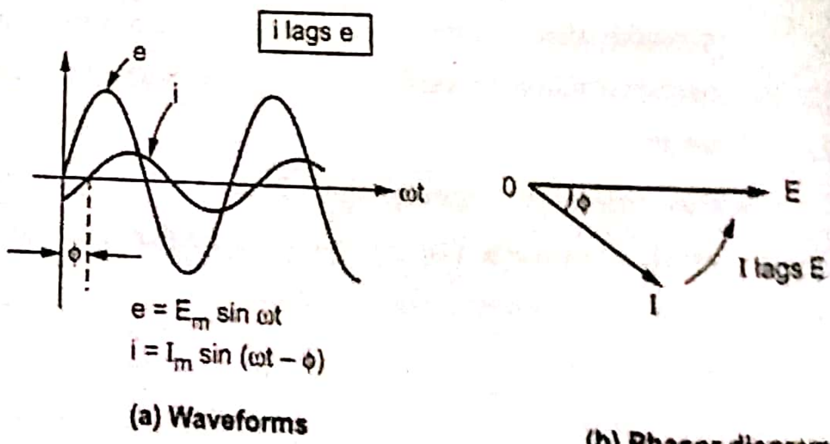

Fig. Q.17.2

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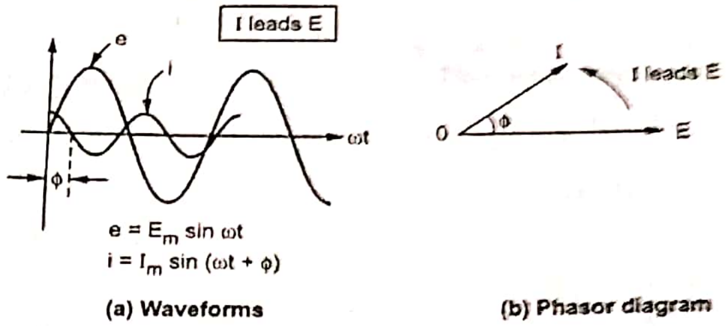

Fig. Q.17.3

Q.18 What is phasor diagram ?

Ans. : The diagram in which different alternating quantities of the same frequency, sinusoidal in nature are represented by individual phasors indicating exact phase interrelationships is known as phasor diagram.

- All phasors have a particular fixed position with respect to each other .
- Phasor diagram can be considered as a still picture of the phasors at a particular instant.

Important Points to Remember

- $e = E_m \sin(\omega t)$ and $i = I_m \sin(\omega t)$ then both e and i are in phase having zero phase difference.
- $e = E_m \sin \omega t$ and $i = I_m \sin(\omega t - \phi)$ then 'i' is said to lag 'e' by angle ϕ.
- $e = E_m \sin \omega t$ and $i = I_m \sin(\omega t + \phi)$ and 'i' is said to **lead** 'e' by angle ϕ.
- For specifying phase difference, the current is specified to be lagging or leading with respect to voltage. The voltage is considered to be reference.

Q.19 A sinusoidal alternating quantity of 50 Hz frequency is having maximum value of current of 100 Amps. Find the time taken by current to attain i) 40 A from origin and ii) 70 A after passing through first positive maximum value.

Ans. : $f = 50$ Hz, $I_m = 100$ A

$$\therefore \quad i(t) = I_m \sin(2\pi f t) = 100 \sin(100\pi t) \text{ A}$$

i) $$i(t) = 40 \text{ A}$$

$$\therefore \quad 40 = 100 \sin(100\pi t)$$

$$\therefore \quad t = \mathbf{1.31\ msec} \quad \text{... Use radian mode}$$

ii) i(t) = 70 A after passing through first positive maximum value.

$$70 = 100 \sin(100\pi t_1)$$

$$\therefore \quad t_1 = 2.4681 \text{ msec}$$

$$T = \frac{1}{f} = 0.02 \text{ sec.}$$

$$\therefore \quad x = \frac{T}{4} - t_1 = 2.5318 \times 10^{-3} \text{ sec}$$

$$\therefore \quad t_2 = \frac{T}{4} + x = \frac{0.02}{4} + 2.5318 \times 10^{-3} = \mathbf{7.5318\ msec}$$

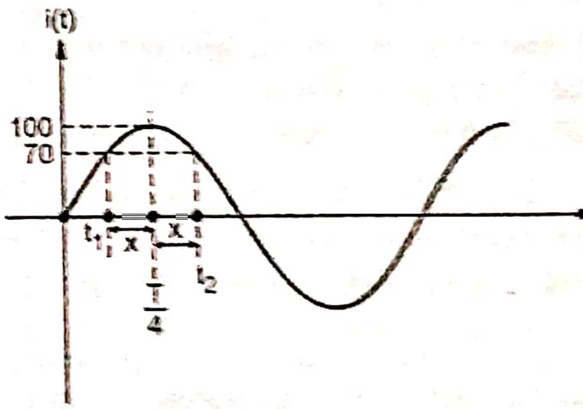

Fig. Q.19.1

Q.20 A 50 Hz alternating current having rms value 10 A has instantaneous value of – 7.07 A at t = 0. Write down the equation for current and sketch the waveform stating all currents and phase angle.

Ans. : $f = 50$ Hz, $I = 10$ A, $i = -7.07$ A at $t = 0$,

$$I_m = \sqrt{2}\, I_{RMS} = \sqrt{2} \times I = \sqrt{2} \times 10 = 14.142 \text{ A},$$
$$\omega = 2\pi f = 314.16 \text{ rad/s}$$

The general equation is, $i = I_m \sin(\omega t + \phi)$

$$\therefore \quad i = 14.142 \sin(314.16\, t + \phi)$$

To find ϕ use $i = -7.07$ A at $t = 0$

$$\therefore \quad -7.07 = 14.142 \sin(\phi) \quad \text{i.e.} \quad \phi = -30° \text{ or } \frac{\pi}{6} \text{ rad}$$

$$\therefore \quad i = 14.142 \sin\left(314.16\, t - \frac{\pi}{6}\right) \text{A}$$

The waveform is shown in the Fig. Q.20.1.

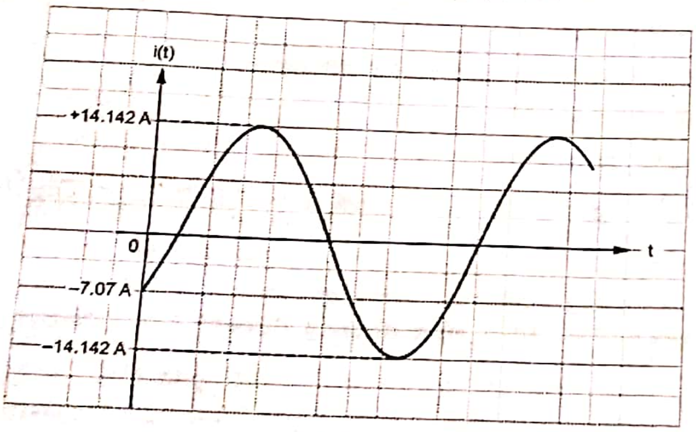

Fig. Q.20.1

2.10 : Mathematical Representation of Phasors

Q.21 How phasors are represented mathematically ? Explain rectangular and polar representations.
[JNTU : Part A, May-07, Dec.-11, Aug.-16 Marks 3]

Ans. : Any phasor can be represented mathematically in two ways,

1) Polar co-ordinate system and 2) Rectangular co-ordinate system.

- Mathematically the polar representation of a phasor is,

$$\text{Polar representation} = r \angle \pm \phi$$

- The polar form of an alternating quantity can be easily obtained from its instantaneous equation directly.

If $\quad e = E_m \sin(\omega t \pm \phi)$ then polar form is,

$$E = E \angle \pm \phi$$

where E is r.m.s. value $= \frac{E_m}{\sqrt{2}}$

- Mathematically the rectangular representation of a phasor is,

$$\text{Rectangular representation} = \pm X \pm j\, Y$$

- To obtain polar form from the instantaneous equation, express the given equation in sine form instead of cosine form.
If $e = E_m \cos(\omega t \pm \phi)$ then express it as,
$e = E_m \sin(\omega t + 90° \pm \phi)$ then
Phase of alternating quantity $= 90° \pm \phi$.

- **Polar to Rectangular Conversion :** Rectangular representation $= r \cos\phi + j\, r \sin\phi$

- **Rectangular to Polar Conversion :** Polar representation $= r \angle \phi = \sqrt{X^2 + Y^2} \angle \tan^{-1}\frac{Y}{X}$

- Use the polar to rectangular **(P → R)** and rectangular to polar **(R → P)** functions available on calculator for the required conversions.

- The r.m.s. value of an alternating quantity exists in its polar form and not in rectangular form. **Thus to find r.m.s. value of an alternating quantity, express it in polar form.**

Important Points to Remember

- Use **rectangular** representation of phasors to perform **addition** and **subtraction**.

Let $\quad P = X_1 + j\, Y_1 \quad$ and $\quad Q = X_2 + j\, Y_2$

- Then analytically while adding P and Q, their X components get added and corresponding Y components get added. Hence the resultant R is,

$$R = P + Q = (X_1 + X_2) + j\,(Y_1 + Y_2)$$

- While subtracting, their X components get subtracted and corresponding Y components get subtracted. Hence the resultant is,

$$R = P - Q = (X_1 - X_2) + j\,(Y_1 - Y_2)$$

- The final result of addition or subtraction then can be expressed in polar form.

Q.22 What is the significance of 'j' notation in the analysis of a.c. circuits ?

[JNTU : Part A, Dec.-16, Marks]

Ans. : In a.c. circuits, the different alternating quantities are represented in rectangular system. The operator 'j' is used to separate real and imaginary parts of rectangular representation of alternating quantity. It indicates the rotation 90° between real and imaginary parts. The positive 'j' indicates anticlockwise rotation while negative 'j' indicates clockwise rotation of real part to get the direction of imaginary part.

Q.23 For the two phasors $A = a_1 + j b_1$ and $B = a_2 + j b_2$, obtain their multiplication and division using polar form of representation.

Ans. : $A = a_1 + jb_1,\ B = a_2 + jb_2$

For multiplication and division use polar co-ordinates.

$$\therefore \quad A = |A| \angle \phi_1 = \sqrt{a_1^2 + b_1^2} \angle \tan^{-1}\frac{b_1}{a_1}$$

$$\therefore \quad B = |B| \angle \phi_2 = \sqrt{a_2^2 + b_2^2} \angle \tan^{-1}\frac{b_2}{a_2}$$

$$\therefore \quad A \times B = |A|\,|B| \angle \phi_1 + \phi_2$$

$$\frac{A}{B} = \frac{|A| \angle \phi_1}{|B| \angle \phi_2} = \frac{|A|}{|B|} \angle \phi_1 - \phi_2$$

Using the values of $|A|, |B|, \phi_1$ and ϕ_2, the multiplication and division of A and B can be obtained.

Important Points to Remember

- For addition and subtraction, use the rectangular co-ordinate system.
- For multiplication and division, use the polar co-ordinate system.

Q.24 Find the result in both rectangular and polar forms, for the following, using complex quantities.

$$\left(30\angle 45° + \frac{1}{3\sqrt{2}\angle -90°}\right) 2\angle 120° + 5\angle -60° - 8\angle 135°$$

[JNTU : May-17, Marks 5]

Ans. : Use rectangular form for addition, subtraction and use polar form for multiplication, division.

Given expression can be expressed as,

$$[21.213 + j21.213 + 0.2357\angle + 90°]\ 2\angle 120° + 2.5 - j4.33 - [-5.657 + j5.657]$$

$$= [21.213 + j21.213 + 0 + j0.2357]\ 2\angle 120° + 8.157 - j9.987$$

$$= [21.213 + j21.4487]\ 2\angle 120° + 8.157 - j9.987$$

$$= (30.167\angle 45.316° \times 2\angle 120°) + 8.157 - j9.987$$

$$= (60.334\angle 165.316°) + 8.157 - j9.987$$

$$= -58.363 + j15.294 + 8.157 - j9.987$$

$$= -50.206 + j5.307 = 50.486\angle 173.966°$$

Q.25 Three currents are represented by $i_1 = 10 \sin \omega t$, $i_2 = 20 \sin\left(\omega t - \frac{\pi}{6}\right)$, $i_3 = 30 \sin\left(\omega t + \frac{\pi}{4}\right)$. Find magnitude and phase angle of resultant current of their addition.

Ans. : $i_1 = 10 \sin \omega t$ i.e. $I_1 = 10\angle 0°$

$i_2 = 20 \sin\left(\omega t - \frac{\pi}{6}\right)$ i.e. $I_2 = 20\angle\frac{-\pi}{6}$

i.e. $20\angle -30°$

$i_3 = 30 \sin\left(\omega t + \frac{\pi}{4}\right)$ i.e. $I_3 = 30\angle\frac{\pi}{4}$

i.e. $30\angle 45°$

$\therefore\ I_1 = 10 + j0$ A, $I_2 = 17.32 - j10$ A,

$I_3 = 21.21 + j21.21$ A

$\therefore\ I_R = I_1 + I_2 + I_3 = 48.53 + j11.21$ A $= 49.81\angle 13°$ A

$\therefore$ Magnitude = 49.81 A, Phase angle = 13°

2.11 : A.C. through Pure Resistance (R)

Q.26 Prove that the voltage and current in purely resistive circuit are in phase. Also derive expression for power.

[JNTU Part B : May-04, Dec.-07, 10, Marks 5]

Ans. : Consider a simple circuit consisting of a pure resistance 'R' ohms connected across a voltage $v = V_m \sin \omega t$.

The circuit is shown in the Fig. Q.26.1.

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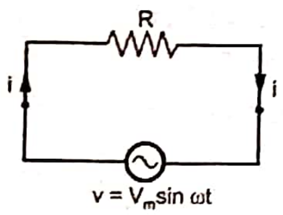

Fig. Q.26.1 Pure resistive circuit

- According to Ohm's law, we can find the equation for the current i as,

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

i.e.

$$i = \left(\frac{V_m}{R}\right) \sin (\omega t)$$

- This is the equation giving instantaneous value of the current.
- Comparing this with standard equation,

$i = I_m \sin (\omega t + \phi)$

$$I_m = \frac{V_m}{R} \quad \text{and} \quad \phi = 0°$$

- So, maximum value of alternating current, i is $I_m = \frac{V_m}{R}$ while as $\phi = 0°$, it indicates that it is in phase with the voltage applied.
- The waveforms of voltage and current and the corresponding phasor diagram is shown in the Fig. Q.26.2 (a) and (b).

Derivation of Power : The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$P = v \times i = V_m \sin(\omega t) \times I_m \sin \omega t$$

$$= V_m I_m \sin^2(\omega t)$$

$$= \frac{V_m I_m}{2} (1 - \cos 2 \omega t)$$

$$\therefore \quad P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos (2 \omega t)$$

- From the above equation, it is clear that the instantaneous power consists of two components,

1) Constant power component $\left(\frac{V_m I_m}{2}\right)$

2) Fluctuating component $\left[\frac{V_m I_m}{2} \cos (2 \omega t)\right]$ having frequency, double the frequency of the applied voltage.

- **The average value of the fluctuating cosine component of double frequency is zero, over one complete cycle.**
- So, average power consumption over one cycle is equal to the constant power component i.e. $\frac{V_m I_m}{2}$ which is half of the peak power $V_m I_m$.

$$\therefore \quad P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} \times I_{rms} \text{ watts}$$

- Generally, r.m.s. values are indicated by capital letters.

$$\therefore \quad P_{av} = V \times I \text{ watts} = I^2 R \text{ watts}$$

2.12 : A.C. through Pure Inductance (L)

Q.27 Prove that in a purely inductive circuit the current lags voltage by 90°. Also prove that its power consumption is zero.

[JNTU : Part B, May-04, 13, Dec.-07, 10, Marks 5]

Ans. : Consider a pure inductance of L henries, connected across a voltage given by the equation, $v = V_m \sin \omega t$ as shown in the Fig. Q.27.1.

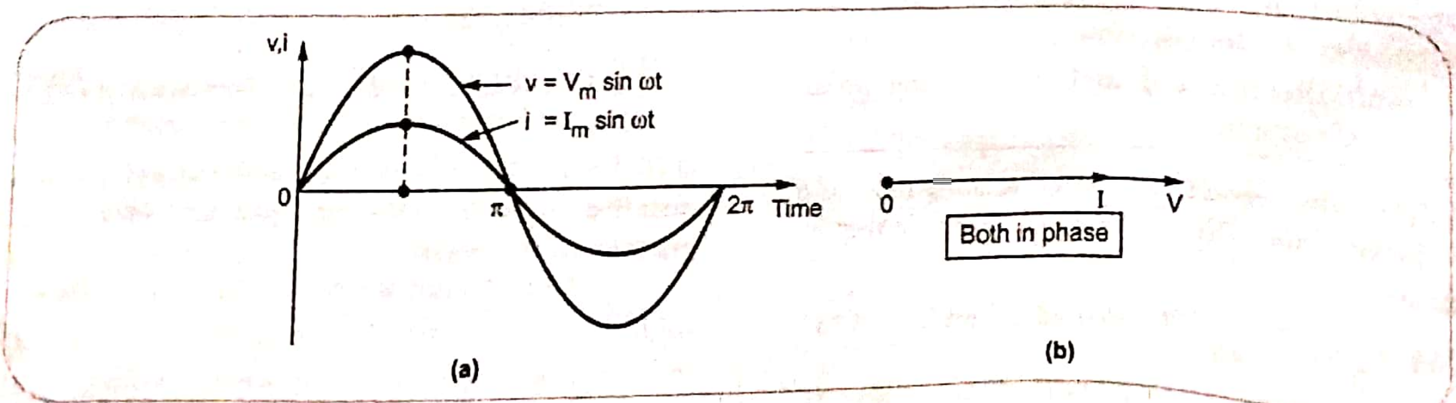

Fig. Q.26.2 A.C. through purely resistive circuit

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- When alternating current 'i' flows through inductance 'L', it sets up an alternating magnetic field around the inductance.
- This changing flux links the coil and due to self inductance, e.m.f. gets induced in the coil. This e.m.f. opposes the applied voltage.
- The self induced e.m.f. in the coil is given by, $e = -L\frac{di}{dt}$.

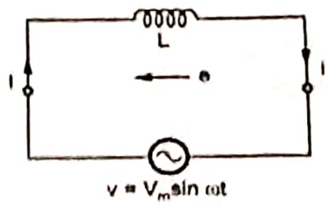

Fig. Q.27.1 Purely inductive circuit

- At all instants, applied voltage, v is equal and opposite to the self induced e.m.f., e.

$$\therefore \quad v = -e = -\left(-L\frac{di}{dt}\right)$$

$$\therefore \quad v = L\frac{di}{dt} \quad \text{i.e.} \quad V_m \sin\omega t = L\frac{di}{dt} \quad \text{i.e.} \quad di = \frac{V_m}{L}\sin\omega t\, dt$$

$$\therefore \quad i = \int di = \int \frac{V_m}{L}\sin\omega t\, dt = \frac{V_m}{L}\left(\frac{-\cos\omega t}{\omega}\right)$$

$$= -\frac{V_m}{\omega L}\sin\left(\frac{\pi}{2}-\omega t\right) \qquad \ldots \cos\omega t = \sin\left(\frac{\pi}{2}-\omega t\right)$$

$$\therefore \quad i = \frac{V_m}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right) \qquad \ldots \sin\left(\frac{\pi}{2}-\omega t\right) = -\sin\left(\omega t-\frac{\pi}{2}\right)$$

$$\therefore \quad \boxed{i = I_m \sin\left(\omega t-\frac{\pi}{2}\right) \text{ where } I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L} \text{ and } X_L = \omega L = 2\pi f L\,\Omega}$$

- The above equation clearly shows that the current is purely sinusoidal and having phase angle of $-\frac{\pi}{2}$ radians i.e. – 90°. This means that **the current lags voltage applied by 90°.**
- The Fig. Q.27.2 shows the waveforms and the corresponding phasor diagram.

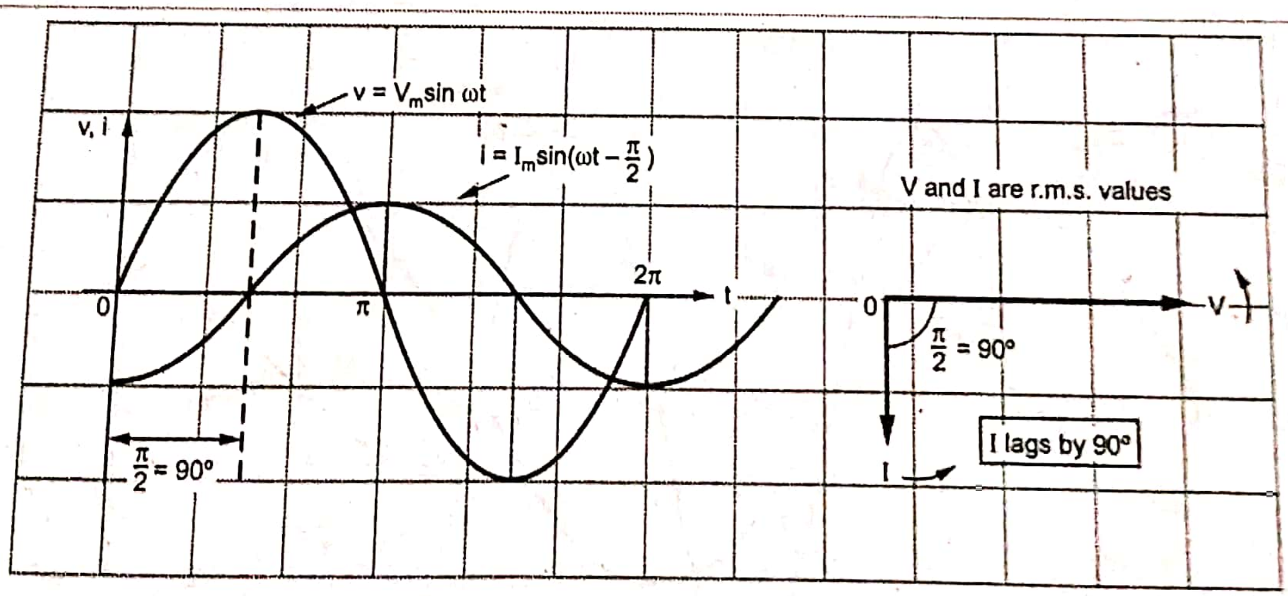

Fig. Q.27.2 A.C. through purely inductive circuit

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- The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$\therefore \quad p = v \times i = V_m \sin \omega t \times I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin(\omega t) \cos(\omega t)$$

$$\text{as } \sin\left(\omega t - \frac{\pi}{2}\right) = -\cos \omega t$$

$$\therefore \quad p = -\frac{V_m I_m}{2} \sin(2\omega t)$$

$$\text{as } 2 \sin \omega t \cos \omega t = \sin 2\omega t$$

- This power curve is a sine curve of frequency double than that of applied voltage.
- The average value of sine curve over a complete cycle is always zero.

$$P_{av} = \int_0^{2\pi} -\frac{V_m I_m}{2} \sin(2\omega t)\, d(\omega t) = 0$$

- Pure inductance never consumes power.

Q.28 Explain the concept of inductive reactance. How it depends on the frequency ?

[JNTU : Part A, May-06,17 Dec.-09, Aug.-17, Marks 3]

Ans. : It is shown that,

$$X_L = \omega L = 2\pi f L \, \Omega$$

- The term, X_L, is called **Inductive Reactance** and is measured in ohms.
- The **inductive reactance** is defined as the opposition offered by the inductance of a circuit to the flow of an alternating sinusoidal current.
- It is measured in ohms and it depends on the frequency of the applied voltage.
- The inductive reactance is directly proportional to the frequency for constant L.

$$X_L \propto f, \quad \text{for constant L}$$

So, graph of X_L Vs f is a straight line passing through the origin as shown in the Fig. Q.28.1.

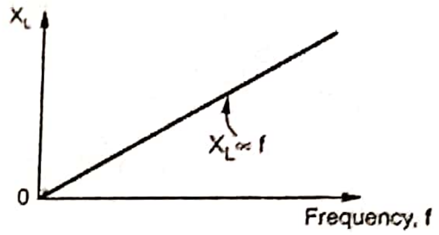

Fig. Q.28.1 X_L Vs f

- *If frequency is zero, which is so for d.c. voltage, the inductive reactance is zero. Therefore, it is said that the inductance offers zero reactance for the d.c. or steady current.*

Q.29 Draw the waveforms of voltage, current and power for a pure inductor when excited by a sinusoidal voltage. [JNTU : Part A, May-19, Marks 3]

Ans. : The waveforms are shown in the Fig. Q.29.1.

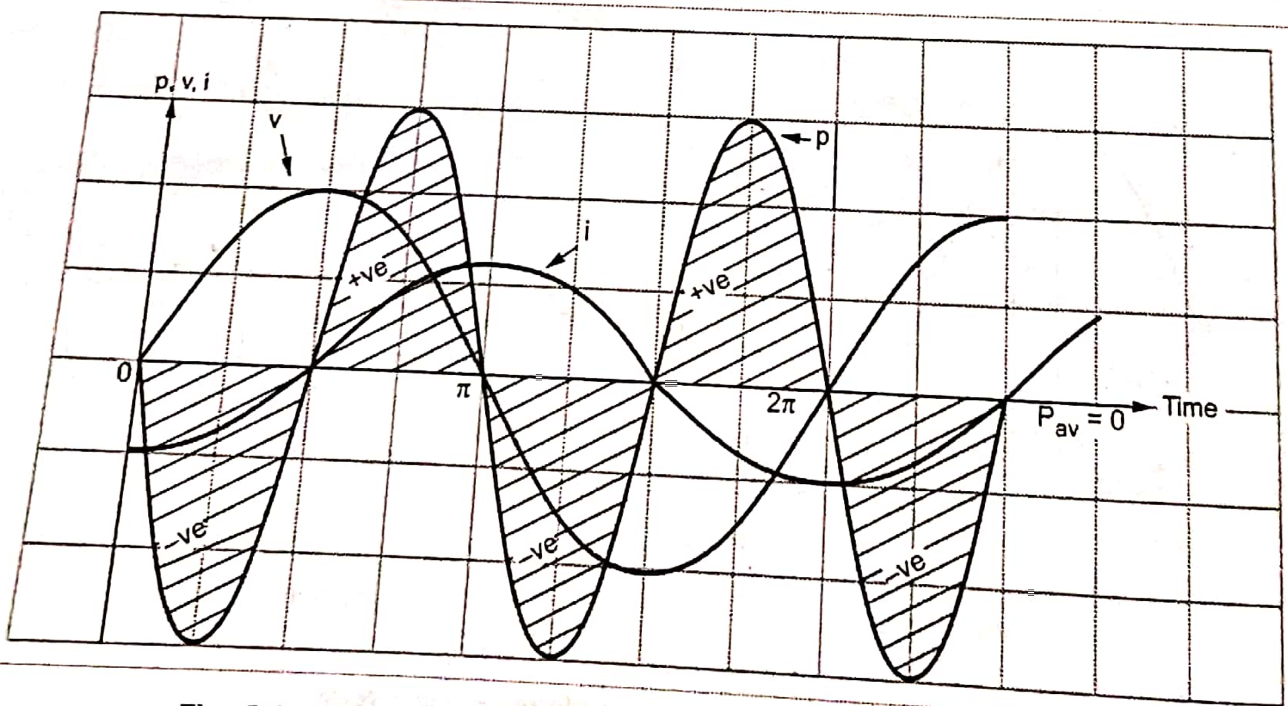

Fig. Q.29.1 Waveforms of voltage, current and power for pure L

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2.13 : A.C. through Pure Capacitance (C)

Q.30 Derive the expressions for current and power for purely capacitive circuit when the voltage applied to it is v (t) = V_m sin ω t. Draw the corresponding phasor diagram. [JNTU : [Part B, May-04, 09, Dec.-07, 10, Marks 5]

OR Prove that in a purely capacitive circuit the current leads voltage by 90°.

[JNTU : Part B, Dec.-14, Marks 5]

Ans. : Consider a pure capacitance of C farads, connected across a voltage given by the equation, $v = V_m \sin \omega t$ as shown in the Fig. Q.30.1.

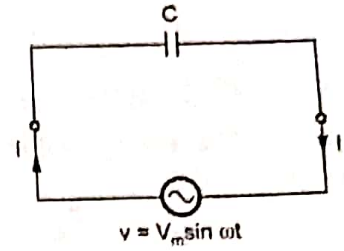

Fig. Q.30.1 Purely capacitive circuit

- The current I charges the capacitor C.
- The instantaneous charge 'q' on the plates of the capacitor is given by,

$$q = C\,v = C\,V_m \sin \omega t$$

- Current is rate of flow of charge.

$$i = \frac{dq}{dt} = \frac{d}{dt}(C\,V_m \sin \omega t) = C\,V_m \frac{d}{dt}(\sin \omega t)$$

$$= C\,V_m\,\omega \cos \omega t$$

$$\therefore \quad i = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right) = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad ...(1)$$

where
$$I_m = \frac{V_m}{X_C} \quad \text{and} \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}\ \Omega$$

- The equation (1) clearly shows that the current is purely sinusoidal and having phase angle of $+\frac{\pi}{2}$ radians i.e. + 90°.
- This means **current leads voltage applied by 90°**. The positive sign indicates leading nature of the current.
- The Fig. Q.30.2 shows waveforms of voltage and current and the corresponding phasor diagram.

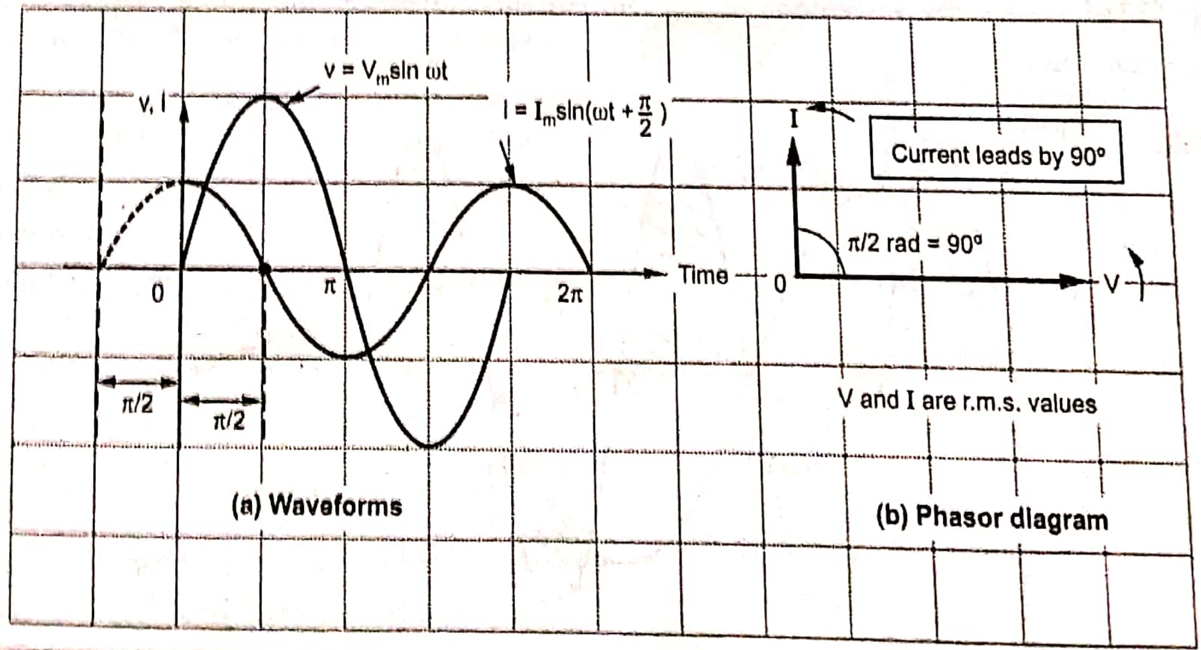

Fig. Q.30.2 A.C. through purely capacitive circuit

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- The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$p = v \times i = V_m \sin(\omega t) \times I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin(\omega t) \cos(\omega t) \qquad \text{as } \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

$$\therefore \quad p = \frac{V_m I_m}{2} \sin(2\omega t) \qquad \text{as } 2 \sin \omega t \cos \omega t = \sin 2\omega t$$

- Thus, power curve is a sine wave of frequency double that of applied voltage.
- **The average value of sine curve over a complete cycle is always zero.**

$$P_{av} = \int_0^{2\pi} \frac{V_m I_m}{2} \sin(2\omega t)\, d(\omega t) = 0$$

- The pure capacitor never consumes power.

Q.31 What is capacitive reactance ? How it varies with frequency ? [JNTU : Part A, May-14, Dec.-08, Marks 2]

Ans. : • The capacitive reactance is defined as the opposition offered by a capacitor to the flow of an alternating sinusoidal current through it.

- It is denoted as X_C and measured in ohms.
- It is mathematically given by,

$$X_C = \frac{1}{2\pi f C} \text{ or } \frac{1}{\omega C} \; \Omega$$

- It can be seen that X_C is inversery proportional to frequency.
- For d.c. having frequency zero ($f = 0$), capacitor offers infinite reactance ($X_C = \infty$) and acts as open circuit. Thus capacitor blocks d.c.

Q.32 Draw the waveforms of voltage, current and power for a pure capacitor when excited by a sinusoidal voltage. [JNTU : Part A, May-13, Aug.-18, Marks 3]

Ans. : The Fig. Q.32.1 shows the waveforms of voltage, current and power for purely capacitive circuit when excited by sinusoidal voltage.

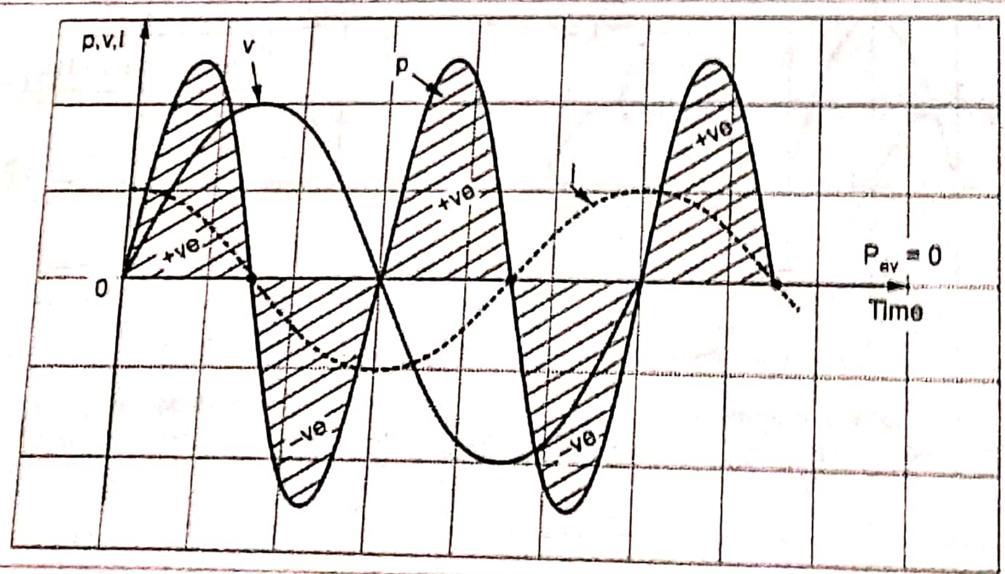

Fig. Q32.1 Waveforms of voltage, current and power for pure C

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2.14 : Impedance

Q.33 Define impedance.
☞ [JNTU : Part A, May-13, Aug.-17, Marks 2]

Ans. : The opposition offered by an electric circuit to the flow of an alternating current is called an impedance. It is denoted by Z.

- It is the ratio of an alternating voltage to an alternating current through the circuit. Impedance is complex and is expressed in polar or rectangular form.

Important Points to Remember

- The impedances for pure R, pure L and pure C are mathematically expressed in rectangular and polar forms as,

$$Z = R + j0 = R\angle 0° \text{ ohms.} \quad \text{... Pure R}$$

$$Z = \frac{V\angle 0°}{I\angle -90°} = \frac{V}{I}\angle 90°$$

$$= X_L\angle 90° = 0 + j\,X_L \text{ ohms} \quad \text{... Pure L}$$

$$Z = \frac{V\angle 0°}{I\angle +90°} = \frac{V}{I}\angle -90°$$

$$= X_C\angle -90° = 0 - j\,X_C \text{ ohms} \quad \text{... Pure C}$$

- The above impedances must be used while solving the problems on a.c. circuits.

2.15 : Power and Power Factor

Q.34 Define power factor. State its importance.
☞ [JNTU : Part A, May-07, Dec.-14, Aug.-17, Marks 2]

Ans. : • The power in d.c. circuits is the product of voltage (V) and the current (I) as V and I are always in phase in d.c. circuits. There is no question of phase difference between the two in d.c. circuits.

- But in a.c. circuits, the voltage (V) and the current (I) may have phase difference between them. The phase angle between the two is denoted as ϕ.
- The power consumption in a.c. circuits depends on the phase angle ϕ between the voltage and the current.

The power in a.c. circuit is given by,
$P = V\,I\cos(\phi)$ where V and I are the r.m.s. values

- Thus the power in a.c. circuit can be defined as the product of the r.m.s. values of voltage and current and cosine of the phase angle between the two.
- The power is measured in watts (W).
- As cosine of the phase angle ϕ determines the power consumption in a.c. circuit, it is called power factor of the circuit.

$$\text{power factor} = \cos(\phi)$$

- The phase angle ϕ is the angle between the supply voltage and the supply current.
- The power factor depends on the elements of the circuit.

The power factor is specified as lagging or leading depending upon the position of current phasor with respect to the voltage phasor.

- Whatever may be the reference phasor, while specifying the nature of the power factor the position of current phasor is considered with respect to voltage phasor.

2.16 : Series R-L Circuit

Q.35 Derive the expression for the current for R-L series circuit when supplied by a voltage $v(t) = V_m \sin \omega t$. Draw the phasor diagram.
☞ [JNTU : Part B, May-05, 07, 11, Dec.-08, 15, Marks 5]

OR Draw the phasor diagram of series R-L circuit.
☞ [JNTU : Part B, May-19, Marks 5]

Ans. : Consider a R-L series circuit as shown in Fig. Q.35.1.

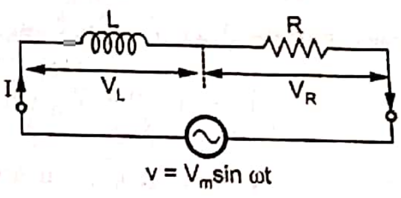

Fig. Q.35.1 Series R-L circuit

- When the circuit draws a current I then there are two voltage drops,

a) Drop across pure resistance,

$$V_R = I \times R$$

b) Drop across pure inductance,

$$V_L = I \times X_L \quad \text{where } X_L = 2\pi f L$$

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I = r.m.s. value of current drawn

V_R , V_L = r.m.s. values of the voltage drops.

The Kirchhoff's voltage law can be applied to the a.c. circuit but only the point to remember is the addition of voltages should be a phasor (vector) addition and no longer algebraic as in case of d.c.

$\therefore$ $\bar{V} = \bar{V}_R + \bar{V}_L = \bar{I}R + \bar{I}X_L$ (Phasor addition)

The phasor diagram for the above case is shown in the Fig. Q.35.2 (a) while the voltage triangle is shown in the Fig. Q.35.2 (b).

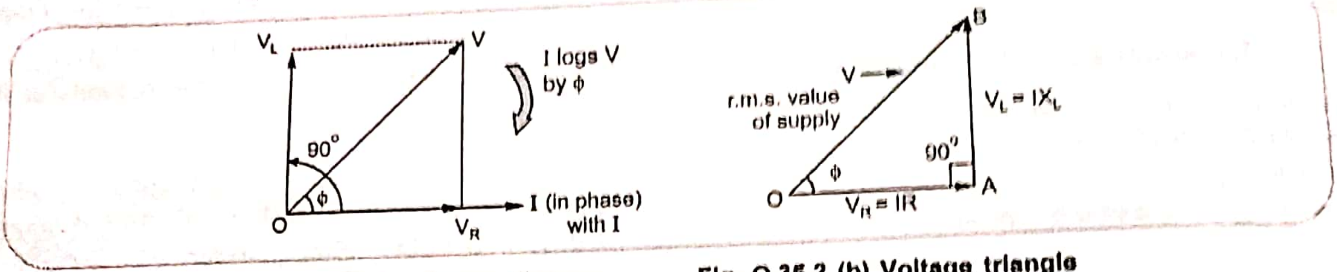

Fig. Q.35.2 (a) Phasor diagram **Fig. Q.35.2 (b) Voltage triangle**

Note that V_R is along the current phasor as for the resistance, voltage and current are in phase. While V_L must be drawn such that current lags V_L by 90°.

From the voltage triangle, shown in the Fig. Q.32.2 (b) we can write,

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (I \times X_L)^2} = I\sqrt{(R)^2 + (X_L)^2} = IZ$$

Where $\boxed{Z = \sqrt{(R)^2 + (X_L)^2}}$ ... **Impedance of the circuit.**

It can be seen that current lags voltage by angle ϕ.

$\therefore$ $\boxed{v(t) = V_m \sin \omega t \text{ and } i(t) = I_m \sin(\omega t - \phi)}$

Q.36 Derive an expression for the average power consumption in series R-L circuit. Draw the waveforms of voltage, current and instantaneous power. **[JNTU : Part B, Dec.-16, Marks 5]**

Ans. : From the given equations of voltage and current, it is clear that the current lags the applied voltage by angle ϕ Thus the circuit is series R-L circuit.

The power is product of instantaneous values of voltage and current,

$$\therefore \quad P = v \times i = V_m \sin \omega t \times I_m \sin(\omega t - \phi)$$

$$= V_m I_m [\sin(\omega t) . \sin(\omega t - \phi)] = V_m I_m \left[\frac{\cos(\phi) - \cos(2\omega t - \phi)}{2}\right]$$

$$= \frac{V_m I_m}{2} \cos\phi - \frac{V_m I_m}{2} \cos(2\omega t - \phi)$$

Now, the second term is cosine term whose average value over a cycle is zero. Hence, average power consumed is,

$$P_{av} = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} . \frac{I_m}{\sqrt{2}} \cos\phi$$

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$\therefore$ $P = V\,I\cos\phi$ watts where V and I are r.m.s. values

- The waveforms of voltage, current and power are shown in the Fig. Q.36.1.

Q.37 Draw power triangle and define active power, reactive power and apparent power.

[JNTU : Part A, May-04, Dec.-12, Marks 3]

Ans. : If we multiply voltage equation of R-L series circuit by current I, we get the power equation.

$$\overline{V}\,\overline{I} = \overline{V_R}\,\overline{I} + \overline{V_L}\,\overline{I}$$

$$= \overline{V\cos\phi I} + \overline{V\sin\phi I}$$

- From this equation, power triangle can be obtained as shown in the Fig. Q.37.1.

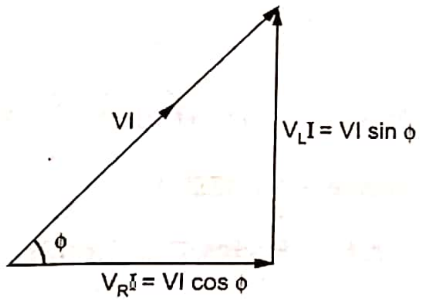

Fig. Q.37.1 Power triangle

- The three sides of this triangle are,

1) VI, 2) VI cos ϕ, 3) VI sin ϕ

1. Apparent Power (S)

It is defined as the product of r.m.s. value of voltage (V) and current (I). It is denoted by S.

$\therefore$ $S = V\,I$ = Apparent power

... measured in volt-amp (VA)

2. Real or True Power (P)

- It is defined as the product of the applied voltage and the active component of the current.
- It is real component of the apparent power. It is measured in unit watts (W) or kilowatts (kW).

P = True or active power = $V\,I\cos\phi$ watts

3. Reactive Power (Q)

- It is defined as product of the applied voltage and the reactive component of the current.
- It is also defined as imaginary component of the apparent power. It is represented by 'Q' and it is measured in unit volt-amp reactive (VAR) or kilovolt-amp reactive (kVAR).

Q = Reactive power = $V\,I\sin\phi$ VAR

Q.38 What is complex power ?

[JNTU : Part A, Dec.-18, Aug.-17, Marks 2]

Ans. : The complex power is the product of the r.m.s. voltage phasor and the complex conjugate of the r.m.s. current phasor. It is expressed in volt-amperes (VA). It is a complex quantity and expressed in rectangular form as,

$$S = P + jQ = V_{rms}\,I^*_{rms}$$

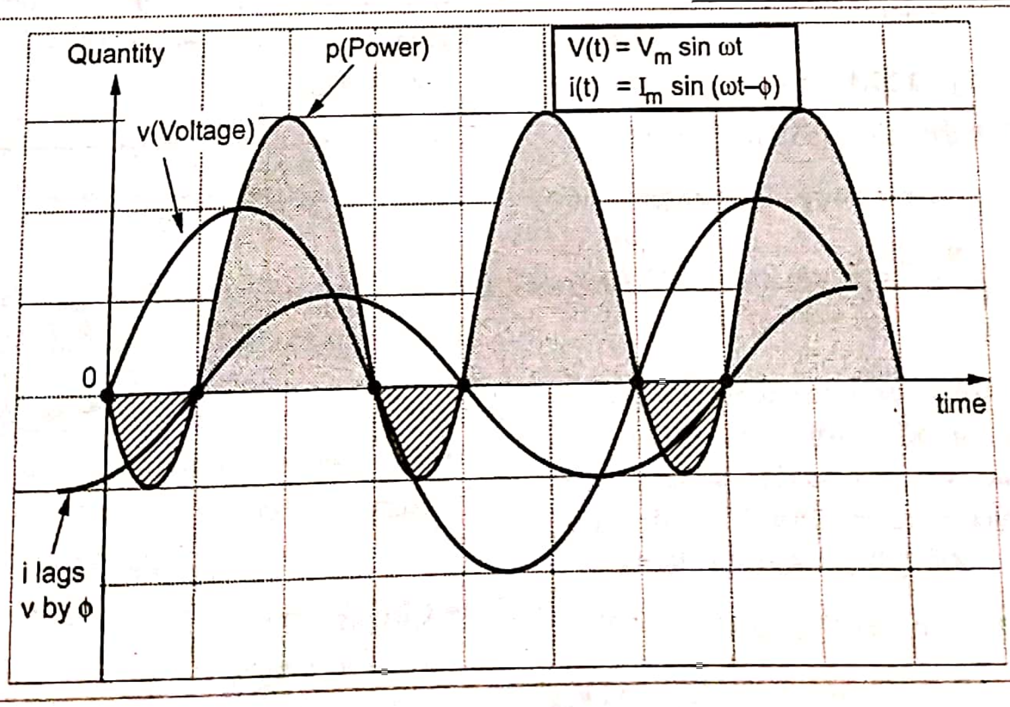

Fig. Q.36.1

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where P = Real power Q = Reactive power

I^*_{rms} = Complex conjugate of I_{rms}

Important Points to Remember

- The nature of power factor is always determined by the position of the current with respect to the voltage.
- If current lags voltage, power factor is said to be lagging. If current leads voltage, power factor is said to be leading.
- For series R-L circuit, $Z = R + jX_L = |Z| \angle \phi$ Ω

$|Z| = \sqrt{R^2 + X_L^2}$,

$\phi = \tan^{-1}\left[\frac{X_L}{R}\right]$, φ is positive for inductive impedance

Power factor = $\cos\phi = \frac{R}{Z}$

Q.39 A coil has a resistance of 4 Ω and an inductance of 9.55 mH. Calculate i) the reactance, ii) the impedance, and iii) the current taken from a 240 V, 50 Hz supply.

[JNTU-H : Part B, May-15, Dec.-08, Aug.-06 Marks 5]

Ans. : R = 4 Ω, L = 9.55 mH, V = 240∠0° V, f = 50 Hz

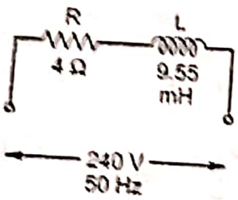


Fig. Q.39.1

i) $X_L = 2\pi fL = 2\pi \times 50 \times 9.55 \times 10^{-3} = 3\,\Omega$

ii) $Z = R + jX_L = 4 + j3\,\Omega = 5 \angle 36.87°\,\Omega$

iii) $I = \frac{V}{Z} = \frac{240\angle 0°}{5\angle 36.87°} = 48 \angle -36.87°$ A

Q.40 A resistance of 60 Ω is connected in series with a pure inductor of 350 mH. The circuit is connected across a 50 Hz sinusoidal supply and the voltage across resistance is 150 V. Calculate the supply volatage. [JNTU : Aug.-18, Marks 5]

Ans. : The circuit is shown in the Fig. Q.40.1.

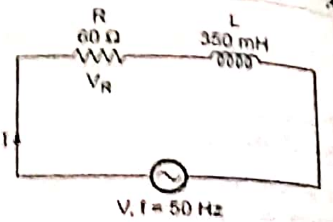


Fig. Q.40.1

$V_R = 150$ V

$V_R = IR$ i.e. $150 = I \times 60$

∴ $I = 2.5$ A (magnitude)

$X_L = 2\pi fL = 2\pi \times 50 \times 350 \times 10^{-3} = 109.956\,\Omega$

∴ $Z = R + jX_L = 60 + j109.956\,\Omega$

$|I| = \frac{V}{|Z|} = \frac{V}{\sqrt{R^2 + X_L^2}}$

∴ $V = 2.5 \times \sqrt{60^2 + (109.956)^2} = 313.1525$ V

∴ Supply voltage = 313.1525 V

2.17 : Series R-C Circuit

Q.41 Derive an expression for current drawn and power consumed by a circuit consisting of 'R' and 'C' connected in series across $v = V_m \sin \omega t$ supply.

[JNTU : Part B, May-05, 07, 11, Dec.-08, 11, 13, Marks]

Ans. : Consider a circuit consisting of pure resistance R-ohms and connected in series with a pure capacitor of C-farads as shown in the Fig. Q.41.1.

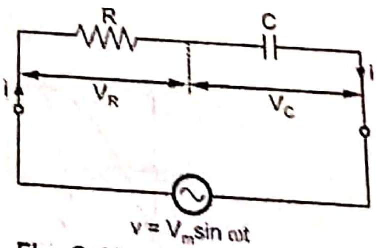


Fig. Q.41.1 Series R-C circuit

- The series combination is connected across a.c. supply given by,

$v = V_m \sin \omega t$

- Circuit draws a current I, then there are two voltage drops,

a) Drop across pure resistance $V_R = I \times R$

b) Drop across pure capacitance $V_C = I \times X_C$

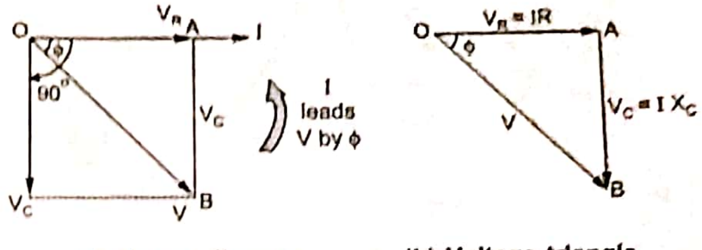
Where $X_C = \frac{1}{2\pi f C}$ and I, V_R, V_C are the r.m.s. values

- The Kirchhoff's voltage law can be applied to get,

$$V = \bar{V_R} + \bar{V_C} = \bar{IR} + \bar{IX_C} \quad \text{... (Phasor Addition)}$$

- The phasor diagram is shwon in the Fig. Q.41.2 (a) while the voltage triangle is shown in the Fig. Q.41.2 (b).

- Note that V_R is along the current phasor as for the resistance, voltage and current are in phase. While V_C must be drawn such that current leads V_C by 90°.



(a) Phasor diagram (b) Voltage triangle

Fig. Q.41.2

- From the voltage triangles,

$$V = \sqrt{(V_R)^2 + (V_C)^2}$$

$$= \sqrt{(IR)^2 + (I X_C)^2} = I\sqrt{(R)^2 + (X_C)^2} = I Z$$

where $Z = \sqrt{(R)^2 + (X_C)^2}$ is the impedance of the circuit.

- It can be seen that current leads voltage by angle φ hence

$$v(t) = V_m \sin \omega t \quad \text{and} \quad i(t) = I_m \sin(\omega t + \phi)$$

- The power is the product of instantaneous values of voltage and current.

$$\therefore \quad P = v \times i = V_m \sin \omega t \times I_m \sin(\omega t + \phi)$$

$$= V_m I_m [\sin(\omega t) \cdot \sin(\omega t + \phi)] = V_m I_m \left[\frac{\cos(-\phi) - \cos(2\omega t + \phi)}{2}\right]$$

$$= \frac{V_m I_m \cos\phi}{2} - \frac{V_m I_m}{2}\cos(2\omega t + \phi) \quad \text{as } \cos(-\phi) = \cos\phi$$

- Now, second term is cosine term whose average value over a cycle is zero. Hence, average power consumed by the circuit is,

$$P_{av} = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$$

$$\therefore \quad P = V I \cos\phi \text{ watts}$$

where V and I are r.m.s. values

Q.42 Draw waveforms of voltage, current and instantaneous power for R-C series circuit. State the expressions for voltage, current and power. ☞ **[JNTU : Part A : May-06, Dec.-14, Marks 2]**

Ans. : The voltage and current for series R-C circuit are given by,

$$v(t) = V_m \sin \omega t \quad \text{and} \quad i(t) = I_m \sin(\omega t + \phi)$$

- While the expression for the instantaneous power is given by,

$$P = \frac{V_m I_m \cos\phi}{2} - \frac{V_m I_m}{2}\cos(2\omega t + \phi)$$

The waveforms for the voltage, current and power for series R-C circuit are shown in the Fig. Q.42.1.

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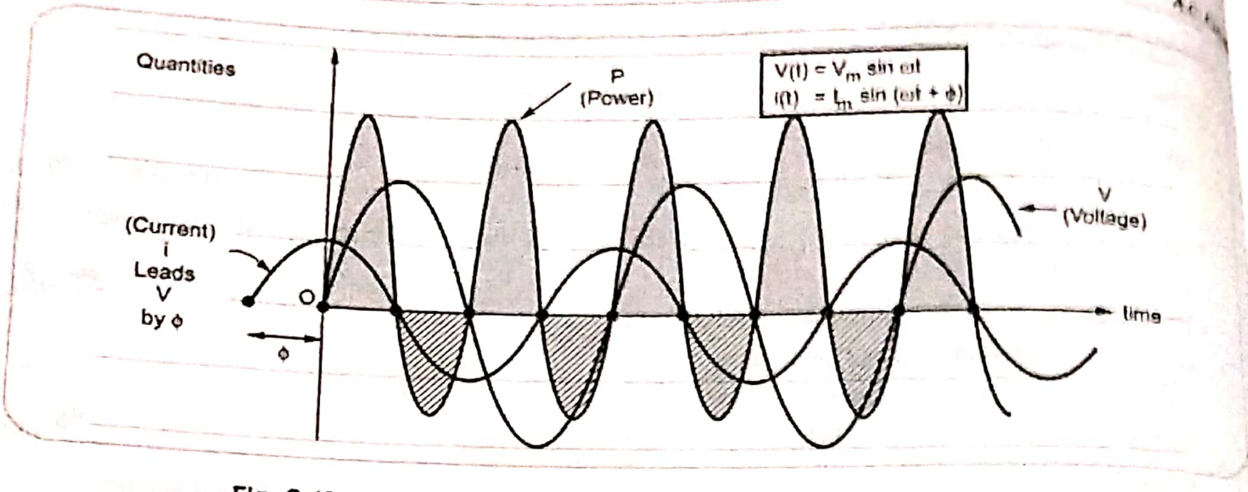


Fig. Q.42.1 Voltage, current and power waveforms for R-C series circuit

Important Points to Remember

- For series R-C circuit, $Z = R - j X_C = |Z| \angle -\phi \ \Omega$
  $|Z| = \sqrt{R^2 + X_C^2}$, $\phi = \tan^{-1}\left[\frac{-X_C}{R}\right]$, ϕ is negative for capacitive impedance.
- The current leads the voltage by angle ϕ in R-C series circuit.

Q.43 A voltage wave e(t) = 141.4 sin (120 t) produces a current, i(t) = 14.14 sin (120 t) + 7.07 cos (120t + 30°) in a circuit. Determine, i) The resultant time expression of the current, ii) The power factor and power delivered by the source. iii) Values of R and C of the circuit. **[JNTU : Part B, Dec.-12, Marks 5]**

Ans. :

i) $i(t) = 14.14 \sin 120\,t + 7.07 \cos(120t + 30°)$

$= 14.14 \sin 120\,t + 7.07 [\cos 120\,t \cos 30° - \sin 120\,t \sin 30°]$

$= 10.605 \sin 120\,t + 6.1228 \cos 120\,t$

$= A \sin(120t + \phi)$

$= A \sin 120\,t \cos\phi + A \cos 120\,t \sin\phi$

Comparing $A \cos\phi = 10.605$, $A \sin\phi = 6.1228$

$\therefore \quad (A \sin\phi)^2 + (A \cos\phi)^2 = A^2 = (10.605)^2 + (6.1228)^2$

$\therefore \quad A = 12.245 \quad \text{and} \quad \tan\phi = \frac{6.1228}{10.605}$

i.e. $\quad \phi = 30°$

$\therefore$ **Resultant current is, $i(t) = 12.245 \sin(120\,t + 30°)$ A**

ii) $E_m = 141.4$ V

and phase 0° from given equation of e(t)

$\therefore \quad E = \frac{141.4}{\sqrt{2}} \angle 0° \text{ V} = 100 \angle 0° \text{ V}, \; I = \frac{12.245}{\sqrt{2}} \angle 30°$

$= 8.6585 \angle 30° \text{ A}$

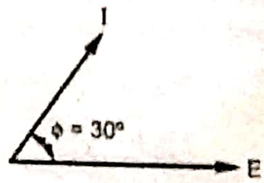
$\therefore$ Power factor angle $= \phi = E \,\hat{}\, I = 30°$

$\therefore$ Power factor $= \cos\phi$

= 0.866 leading

$\therefore \; P = EI\cos\phi = 100 \times 8.6585 \times \cos 30°$

= 749.85 W



iii) $Z = \frac{E}{I} = \frac{100\angle 0°}{8.6585 \angle 30°} = 11.5493 \angle -30° \ \Omega$

$= 10 - j5.7746 \ \Omega$

Comparing with, $Z = R - jX_C$

$R = 10 \ \Omega, \; X_C = 5.7746 = \frac{1}{2\pi f C} = \frac{1}{\omega C}$

and $\omega = 120$ rad/s from given equations

$$\therefore \quad C = \frac{1}{120\times 5.7746} = 1.4431 \text{ mF}$$

Note that $e(t) = E_m \sin \omega t$

$$= 141.4 \sin(120\,t) \text{ V hence } \omega = 120 \text{ rad/sec.}$$

Q.44 Develop the phasor diagram for a series RC circuit with R = 10 Ω and C = 10 μF and excited with a 1 - ϕ 230 V ac supply.

☞ [JNTU : Part B, Dec.-16, Jan-10, 11, Marks 5]

Ans. : $V = 230$ V, $R = 10\Omega$, $C = 10\,\mu F$, $f = 50$ Hz

$$\therefore \quad Z = R - jX_C$$

where $$X_C = \frac{1}{2\pi fC} = 318.31\ \Omega$$

$$\therefore \quad Z = 10 - j318.31\ \Omega$$

$$= 318.46 \angle -88.2°\ \Omega$$

$$\therefore \quad I = \frac{V}{Z}$$

$$= \frac{230\angle 0°}{318.46\angle -88.2°}$$

$$= 0.722\angle +88.2°\text{ A}$$

$$\therefore \quad \overline{V}_R = IR = 0.722\angle 88.2° \times 10\angle 0°$$

$$= 7.22\angle 88.2°\text{ V}$$

$$\overline{V}_C = I(-jX_C)$$

$$= 0.722\angle 88.2° \times 318.31\angle -90°$$

$$= 229.82\angle -1.8°\text{ V}$$

$$\overline{V} = \overline{V}_R + \overline{V}_C$$

Hence the phaser diagram is as shown in the Fig. Q.44.1.

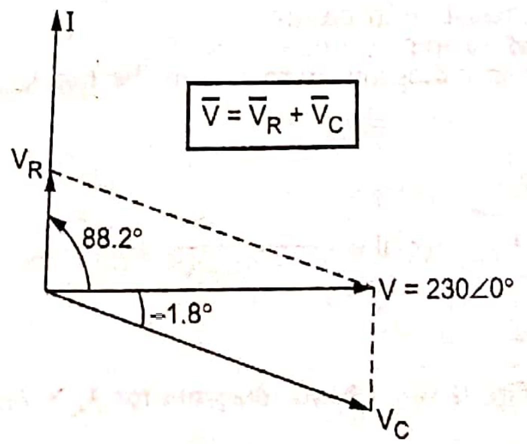


Fig. Q.44.1

Q.45 A voltage v = 100 sin 314 t is applied to a circuit consisting of a 25 ohm resistor and an 80 μF capacitor in series. Determine i) Peak value of current ii) Power factor iii) Total power consumed by the circuit.

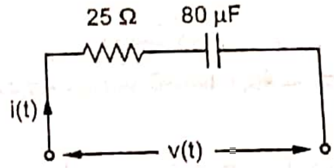


Fig. Q.45.1

Ans. :

i) Comparing given voltage with $V_m \sin \omega t$,

$$V_m = 100 \text{ V}, \ \omega = 314 \text{ rad/s}$$

$$\therefore \quad X_C = \frac{1}{\omega C} = \frac{1}{314\times 80\times 10^{-6}} = 39.8089\ \Omega$$

$$\therefore \quad Z = R - jX_C = 25 - j\,39.8089$$

$$= 47\angle -57.87°\ \Omega$$

i) $$I_m = \frac{V_m}{|Z|} = \frac{100}{47} = \mathbf{2.1276\ A} \quad \text{... Peak value}$$

ii) $$\cos\phi = \frac{R}{Z} = \frac{25}{47} = \mathbf{0.5319\ leading}$$

... Power factor

iii) $$P = V\,I\cos\phi = \frac{V_m}{\sqrt{2}}\times\frac{I_m}{\sqrt{2}}\cos\phi$$

$$= \frac{100\times 2.1276}{2}\times 0.5319 = \mathbf{56.5851\ W}$$

2.18 : Series R-L-C Circuit

Q.46 Sketch and explain the phasor diagram of RLC series circuit for i) $X_C > X_L$ ii) $X_C < X_L$ iii) $X_C = X_L$

☞[JNTU : Part B, May-04, Aug.-04, Dec.-09, 12, Marks 5]

Ans. : Consider a circuit consisting of resistance R ohms pure inductance L henries and capacitance C farads connected in series with each other across a.c supply. The circuit is shown in the Fig. Q.46.1.

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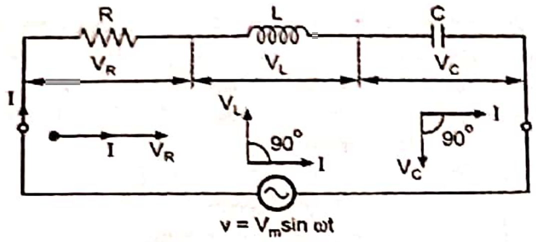

Fig. Q.46.1 R-L-C series circuit

- The a.c. supply is given by, $v = V_m \sin \omega t$.
- The circuit draws a current I.
- Due to current I, there are different voltage drops across R, L and C which are given by,

∴

a) Drop across resistance R is $V_R = I R$
b) Drop across inductance L is $V_L = I X_L$
c) Drop across capacitance C is $V_C = I X_C$

- According to Kirchhoff's laws, we can write,

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C \quad \text{... Phasor addition}$$

Following are the steps to draw the phasor diagram :

1) Take current as reference.
2) V_R is in phase with I.
3) V_L leads current I by 90°.
4) V_C lags current I by 90°.
5) Obtain the resultant of V_L and V_C. Both V_L and V_C are in phase opposition (180° out of phase).
6) Add that with V_R by law of parallelogram to get the supply voltage.

Case 1 : $X_L > X_C$

- When $X_L > X_C$, obviously, $I X_L$ i.e. V_L is greater than $I X_C$ i.e. V_C. So, resultant of V_L and V_C will be directed towards V_L i.e. leading current I.
- The phasor sum of V_R and $(V_L - V_C)$ gives the resultant supply voltage, V. This is shown in the Fig. Q.46.2.
- The circuit is inductive in nature and current lags the voltage.

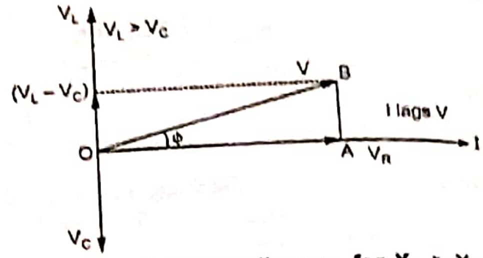

Fig. Q.46.2 Phasor diagram for $X_L > X_C$

Case 2 : $X_L < X_C$

- When $X_L < X_C$, obviously, $I X_L$ i.e. V_L is less than $I X_C$ i.e. V_C. So, the resultant of V_L and V_C will be directed towards V_C.
- The phasor sum of V_R and $(V_C - V_L)$ gives the resultant supply voltage V. This is shown in the Fig. Q.46.3.
- The circuit is capacitive in nature and current leads the voltage.

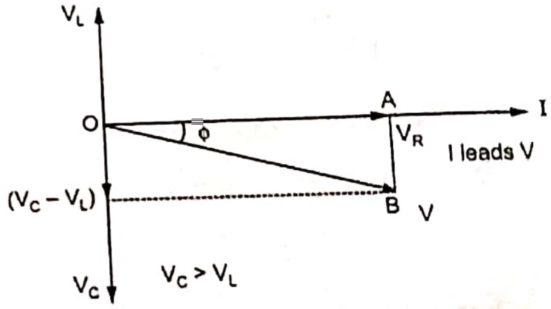

Fig. Q.46.3 Phasor diagram for $X_L < X_C$

Case 3 : $X_L = X_C$

- When $X_L = X_C$, obviously, $V_L = V_C$.
- So, V_L and V_C will cancel each other and their resultant is zero.
- Thus $V_R = V$ in such case and overall circuit is purely resistive in nature.
- The phasor diagram is shown in the Fig. Q.46.4.

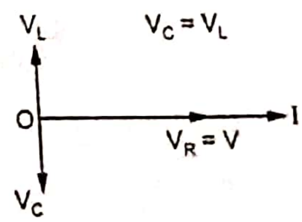

Fig. Q.46.4 Phasor diagram for $X_L = X_C$

Summary of RLC Circuits :

Sr. No.	Circuit	Impedance (Z) Polar	Impedance (Z) Rectangular	ϕ	p.f. $\cos\phi$	Remark
1.	Pure R	$R\angle 0°\ \Omega$	$R + j0\ \Omega$	0°	1	Unity p.f.
2.	Pure L	$X_L \angle 90°\ \Omega$	$0 + j X_L\ \Omega$	90°	0	Zero lagging
3.	Pure C	$X_C \angle -90°\ \Omega$	$0 - j X_C\ \Omega$	– 90°	0	Zero leading
4.	Series RL	$\lvert Z\rvert \angle +\phi°\ \Omega$	$R + j X_L\ \Omega$	$0° \angle \phi \angle 90°$	$\cos\phi$	Lagging
5.	Series RC	$\lvert Z\rvert \angle -\phi°\ \Omega$	$R - jX_C\ \Omega$	$-90° \angle \phi \angle 0°$	$\cos\phi$	Leading
6.	Series RLC	$\lvert Z\rvert \angle \pm \phi°\ \Omega$	$R + j X\ \Omega$ $X = X_L - X_C$	ϕ	$\cos\phi$	$X_L > X_C$ Lagging $X_L < X_C$ Leading $X_L = X_C$ Unity

Q.47 Find the impedance of series R-L-C circuit with R = 100 Ω, X_L = 50 Ω and X_C = 20 Ω

[JNTU : Aug.-17, Marks 4]

Ans. : R = 100 Ω, X_L = 50 Ω, X_C = 20 Ω

$$\therefore \quad Z = R + jX_L - jX_C = 100 + j50 - j20$$

$$= 100 + j\,30\ \Omega$$

$$= 104.403 \angle 16.7°\ \Omega$$

Q.48 An inductance of 0.5 H, a resistance of 5 ohms, and a capaciatance of 8 μF are in series across a 220 V, 50 Hz, AC supply. Find the voltage across each element and total current supplied by the supply and draw the phasor diagram for the circuit.

[JNTU : May-18, Marks 5]

Ans. : The circuit is shown in the Fig. Q.48.1

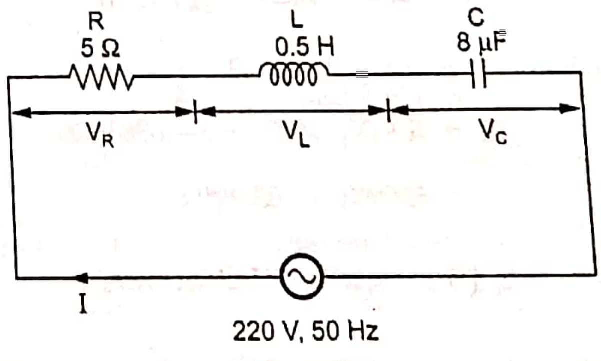

Fig. Q.48.1

$$X_L = 2\pi f L = 157.08\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = 397.89\ \Omega$$

$$Z = R + jX_L - jX_C$$

$$= 5 + j157.08 - j397.89$$

$$\therefore \quad Z = 5 - j240.81\ \Omega$$

$$= 240.862 \angle -88.81°\ \Omega$$

Assuming voltage as reference, V = 220 ∠ 0° V

$$\therefore \quad I = \frac{V}{Z} = \frac{220\angle 0°}{240.862\angle -88.81°} = 0.9134 \angle 88.81°\ \text{A}$$

$$\therefore \quad V_R = I(R\angle 0°)$$

$$= 0.9134 \angle 88.81° \times 5\angle 0°$$

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$= 4.567 \angle 88.81° \text{ V}$

$\therefore \quad V_L = I(X_L \angle 90°)$

$= 0.9134 \angle 88.81° \times 157.08 \angle 90°$

$= 143.47 \angle 178.81° \text{ V}$

$\therefore \quad V_C = I(X_C \angle -90°)$

$= 0.9134 \angle 88.81° \times 397.89 \angle -90°$

$= 363.432 \angle -1.19° \text{ V}$

The phasor diagram is shown in the Fig. Q.48.1 (a).

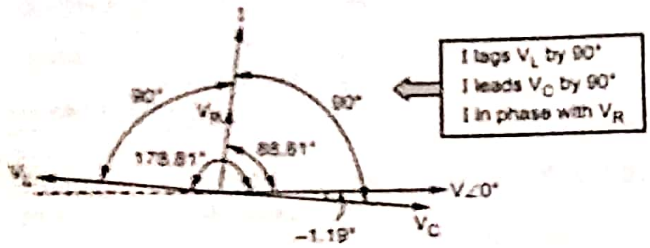


Fig. Q.48.1 (a)

Q.49 Determine the impedance of series RLC circuit with R = 5 Ω, L = 2 mH and C = 5 nF with an applied voltage of v(t) = 10 sin (314 t) [JNTU-H : Part A, Dec.-2016, Marks 3]

Ans. : Compare v(t) with $V_m \sin(\omega t)$, $\omega = 314$ rad/sec.

$$\therefore \quad X_L = 2\pi f L = \omega L = 314 \times 2 \times 10^{-3} = 0.628\,\Omega$$

$$\therefore \quad X_C = \frac{1}{2\pi f L} = \frac{1}{\omega C} = \frac{1}{314 \times 5 \times 10^{-9}}$$

$$= 636.943 \times 10^3\,\Omega$$

$$\therefore \quad Z = R + jX_L - jX_C = 5 - j636.942 \times 10^3\,\Omega$$

$$= 636942 \angle -89.99°\,\Omega$$

2.19 : Parallel RLC Circuits

Q.50 What is admittance ? Which are its two components ? State their units. How the admittance is expressed in rectangular and polar form. [JNTU : Part A, Dec.-09, 11, May-08, Marks 3]

Ans. : *Admittance is defined as the reciprocal of the impedance. It is denoted by Y and is measured in unit siemens or mho.*

- Consider an impedance given as, $Z = R \pm jX$
- Positive sign for inductive and negative for capacitive circuit.

$$\text{Admittance} \quad Y = \frac{1}{Z} = \frac{1}{R \pm jX}$$

- Rationalising the above expression,

$$Y = \frac{R \mp jX}{(R \pm jX)(R \mp jX)} = \frac{R \mp jX}{R^2 + X^2}$$

$$= \left(\frac{R}{R^2 + X^2}\right) \mp j\left(\frac{X}{R^2 + X^2}\right) = \frac{R}{Z^2} \mp j\frac{X}{Z^2}$$

$$\therefore \quad Y = G \mp jB \text{ where } G = \text{Conductance} = \frac{R}{Z^2},$$

$$B = \text{Susceptance} = \frac{X}{Z^2}$$

Conductance (G) :

- It is defined as the ratio of the resistance to the square of the impedance. It is measured in the unit siemens.

Susceptance (B) :

- It is defined as the ratio of the reactance to the square of the impedance. It is measured in the unit siemens.

The susceptance is said to be inductive (B_L) if its sign is negative. The susceptance is said to be capacitive (B_C) if its sign is positive.

- Admittance in polar form is expressed as,

$$Y = G + jB = |Y| \angle \phi \text{ siemens or mho}$$

$$|Y| = \sqrt{G^2 + B^2}, \quad \phi = \tan^{-1}\frac{B}{G}$$

B is negative if inductive and B is positive if capacitive

Q.51 Explain the admittance method to solve the parallel a.c. circuits. [JNTU : Part B, Dec.-14, May-10, Marks 5]

Ans. : • The various steps to solve the parallel circuit by admittance method are,

Step 1 : Calculate the admittance of each branch from the respective impedance.

$$Y_1 = \frac{1}{Z_1}, \; Y_2 = \frac{1}{Z_2}, \; Y_3 = \frac{1}{Z_3} \; ......$$

Step 2 : Convert all the admittances to the respective rectangular form.

Step 3 : Calculate the equivalent admittance of the circuit by adding the individual admittances of the branches.

$$Y_{eq} = Y_1 + Y_2 + Y_3 ..... = G_{eq} + B_{eq} = |Y| \angle \phi$$

Step 4 : The total current drawn from the supply is then given by,

$$I_T = V \times Y_{eq}$$

Step 5 : The individual branch currents can be obtained as,

$$I_1 = V \times Y_1, \quad I_2 = V \times Y_2, \quad I_3 = V \times Y_3 .....$$

- It can be crosschecked that the vector addition of all the above currents gives the total current calculated in step 4.

Step 6 : The angle between V and I_T is the power factor angle ϕ. The cosine of this angle is the power factor of the circuit. The power factor of the circuit can also be obtained as,

$$\cos \phi = \frac{G_{eq}}{Y_{eq}}$$

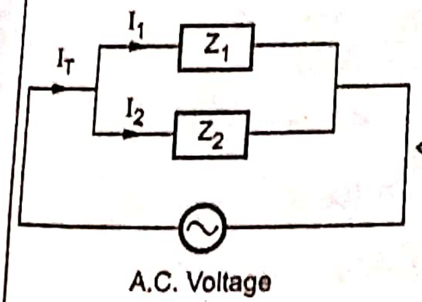
- The nature of the power factor is to be decided from the sign of B_{eq}. If it is negative power factor is lagging while if it is positive the power factor is leading.

Step 7 : Voltage must be taken as reference phasor as it is common to all branches to draw the phasor diagram.

Important Point to Remember

Current division in two parallel impedances :

- Similar to the current division in two parallel resistance, the current division in two parallel impedances is given by,



$$I_1 = I_T \times \frac{Z_2}{Z_1 + Z_2}$$

$$I_2 = I_T \times \frac{Z_1}{Z_1 + Z_2}$$

Fig. 2.19.1

It can be cross checked that $\bar{I}_T = \bar{I}_1 + \bar{I}_2$ (phasor sum)

Q.52 If admittance of a series circuit is (0.010 + j 0.004)S. Determine the values of circuit components for the frequency value of 50 Hz ? [JNTU : Part B, May-19, Marks 3]

Ans. :

Given : Y = 0.010 + j 0.004 = 0.01077 ∠ 21.801° S

$$\therefore \quad Z = \frac{1}{Y} = \frac{1}{0.01077 \angle 21.801^\circ}$$

$$= 92.8505 \angle -21.801^\circ \ \Omega$$

$$= 86.209 - j\,34.484 \ \Omega$$

$$= R - jX_C$$

$$\therefore \quad R = 86.209 \ \Omega, \quad X_C = 34.484 \ \Omega = \frac{1}{2\pi f C}$$

$$\therefore \quad C = \frac{1}{2\pi \times 50 \times 34.484} = 92.306 \ \mu F$$

Q.53 The circuit having two impedances in parallel of $Z_1 = 8 + j15 \Omega$ and $Z_2 = 6 - j8 \ \Omega$, connected to a single phase a.c. supply and the current drawn is 10 A. Find each branch current, both in magnitude and phase and also the supply voltage.

Ans. : The circuit is shown in the Fig. Q.53.1.

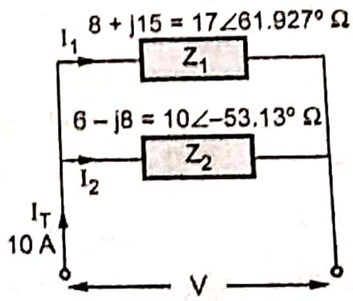


Fig. Q.53.1

Assume total current is reference.

$$\therefore \quad I_T = 10 \angle 0^\circ \text{ A}$$

Using current distribution rule,

$$\bar{I}_1 = I_T \times \frac{Z_2}{Z_1 + Z_2} = 10 \angle 0^\circ \times \frac{10 \angle -53.13^\circ}{8 + j15 + 6 - j8}$$

$$= \frac{100 \angle -53.13^\circ}{(14 + j7)} = \frac{100 \angle -53.13^\circ}{15.6524 \angle 26.565^\circ}$$

$$= 6.3887 \angle -79.695^\circ \text{ A}$$

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$$I_2 = I_T \times \frac{Z_1}{Z_1 + Z_2} = 10\angle 0^\circ \times \frac{17\angle 61.927^\circ}{(14 + j7)}$$

$$= \frac{10\angle 0^\circ \times 17\angle 61.927^\circ}{15.6524\angle 26.565^\circ} = 10.861\angle 35.362^\circ \text{ A}$$

Key Point : Check that $\bar{I}_T = \bar{I}_1 + \bar{I}_2$

$\therefore$ $V = \overline{I_1 Z_1} = \overline{I_2 Z_2}$

$= 6.3887\angle -79.695^\circ \times 17\angle 61.927^\circ$

$= 108.6079\angle -17.768^\circ$ V

... Supply voltage

Q.54 In the following circuit shown in Fig. Q.54.1, the effective voltage between points A and B is 25 volts. Find the corresponding effective values of V and I_T. **[JNTU : May-18, Marks 5]**

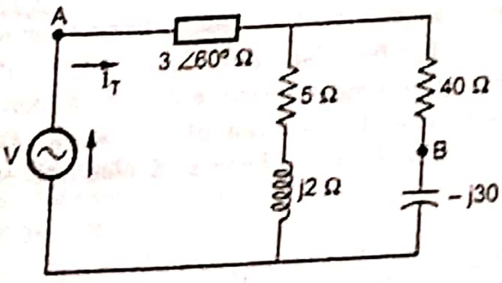

Fig. Q.54.1

Ans. : Two branches are in parallel.

- By current division in parallel circuit, current through branch 40 – j30 Ω is,

$$I = I_T \times \frac{5 + j2}{(5 + j2 + 40 - j30)}$$

$$= I_T \times \frac{5.385\angle 21.8^\circ}{53\angle -31.89^\circ}$$

$$= I_T \times 0.1016\angle 53.69^\circ$$

$\therefore$ $V_{AB} = I_T \times 3\angle 60^\circ + I \times 40$

$= I_T \times 3\angle 60^\circ + I_T \times 0.1016\angle 53.69^\circ \times 40$

$= I_T [1.5 + j2.598 + 2.4065 + j3.2748]$

$= I_T [7.053\angle 56.368^\circ]$ but $V_{AB} = 25$ V

$\therefore$ $|V_{AB}| = 25 = 7.053\, I_T$

$I_T = 3.5446$ A ...Magnitude

$Z_{eq} = 3\angle 60^\circ + [(5 + j2) \| (40 - j30)]$

$$= 3\angle 60^\circ + \left\{\frac{5.385\angle 21.8^\circ \times 50\angle -36.87^\circ}{5 + j2 + 40 - j30}\right\}$$

$= (1.5 + j2.598) + (5.08\angle 16.821^\circ)$

$= 1.5 + j2.598 + 4.863 + j1.47$

$= 6.363 + j4.068\ \Omega$

$= 7.552\angle 32.59^\circ\ \Omega$

$\therefore$ $|Z_{eq}| = 7.552\ \Omega,\ |I_T| = 3.5446$

$V = |I_T| \times |Z_{eq}| = 26.77$ V ...magnitude

Q.55 An impedance $Z_1 = (6 + j8)\ \Omega$ is connected in series with a parallel combination of impedances $Z_2 = (10 + j6)\ \Omega$, $Z_3 = (8 - j10)\ \Omega$ and connected to a 300 V, 50 Hz, supply. Find the total active power, reactive power and power factor of the circuit. **[JNTU : May-19, Marks 5]**

Ans. : The circuit is shown in the Fig. Q.55.1.

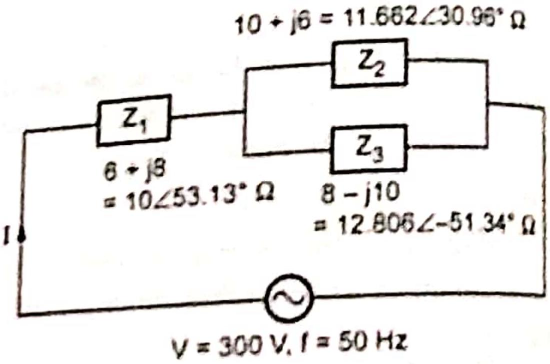

Fig. Q.55.1

$$Z_2 \| Z_3 = \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$= \frac{11.662\angle 30.96^\circ \times 12.806\angle -51.34^\circ}{10 + j6 + 8 - j10}$$

$$= \frac{149.343\angle -20.38^\circ}{18.439\angle -12.528^\circ}$$

$= 8.099\angle -7.852^\circ\ \Omega$

$= 8.023 - j1.106\ \Omega$

$\therefore$ $Z_T = Z_1 + (Z_2 \| Z_3)$

$= 6 + j8 + 8.023 - j1.106$

$= 14.023 + j6.894\ \Omega$

$= 15.626\angle 26.18^\circ\ \Omega$

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$\therefore \quad I = \frac{V}{Z_T} = \frac{300 \angle 0^\circ}{15.626 \angle 26.18^\circ}$

$= 19.198 \angle -26.18^\circ$ A

$\therefore \quad P$ = Total active power = VI cos ϕ

$= 300 \times 19.198 \times \cos(26.18^\circ)$

= **5168.55 W**

$\therefore \quad Q$ = Total reactive power = VI sin ϕ

$= 300 \times 19.198 \times \sin(26.18^\circ)$

= **2541 VAR**

$\therefore \quad \cos\phi$ = Power factor = $\cos(26.18^\circ)$

= **0.8974 lagging**

2.20 : Three Phase Balanced Circuits

Important Points to Remember

Three Phase A.C. Circuits

- In a three phase supply system, there are three voltages with a same magnitude and frequency but having a phase difference of 120° between them.
- The equations for the three phase voltages are :

$$e_R = E_m \sin(\omega t)$$

$$e_Y = E_m \sin(\omega t - 120^\circ)$$

$$e_B = E_m \sin(\omega t - 240^\circ)$$

$$= E_m \sin(\omega t + 120^\circ)$$

- The phasor diagram of these voltages can be shown as in the Fig. 2.20.1

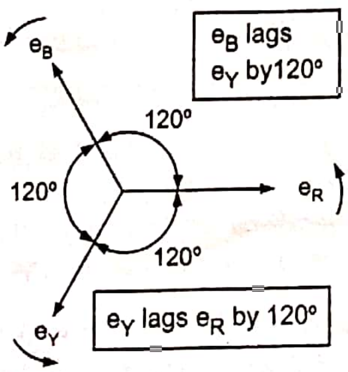

Fig. 2.20.1 Phasor diagram

If we add three voltages vectorially, it can be observed that the sum of the three voltages at any instant is zero.

2.21 : Advantages of Three Phase System

Q.56 State the various advantages of three phase system over single phase system. [JNTU : Part A, Marks 3]

Ans. : A three phase system has following advantages over single phase system :

1) The output of three phase machine is always greater than single phase machine of same size, approximately 1.5 times. So for a given size and voltage a three phase alternator occupies less space and has less cost too than single phase having same rating.

2) For a transmission and distribution, three phase system needs less copper or less conducting material than single phase system for given volt amperes and voltage rating so transmission becomes very much economical.

3) It is possible to produce rotating magnetic field with stationary coils by using three phase system. Hence three phase motors are self starting.

4) In single phase system, the instantaneous power is a function of time and hence fluctuates w.r.t. time. This fluctuating power causes considerable vibrations in single phase motors. Hence performance of single phase motors is poor. While instantaneous power in symmetrical three phase system is constant.

5) Three phase system give steady output.

6) Single phase supply can be obtained from three phase but three phase cannot be obtained from single phase.

7) Power factor of single phase motor is poor than three phase motors of same rating.

8) For converting machines like rectifiers, the d.c. output voltage becomes smoother if number of phases are increased.

2.22 : Phase Sequence

Q.57 What is the meaning of phase sequence ? How it can be changed ? [JNTU : Part A, Marks 3]

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Ans. : • The sequence in which the voltages in three phases reach their maximum positive values is called **phase sequence**. Generally the phase sequence is R-Y-B.

- The phase sequence is important in determining direction of rotation of a.c. motors, parallel operation of alternators etc.
- The phase sequence of a three phase system can be changed by interchanging any two terminals out of R, Y and B.

2.23 : Three Phase Supply Connections

Important Points to Remember

- The two types of three phase supply connections are,
  1. Star connection 2. Delta connection
- The Fig. 2.23.1 shows the star connection of supply lines.

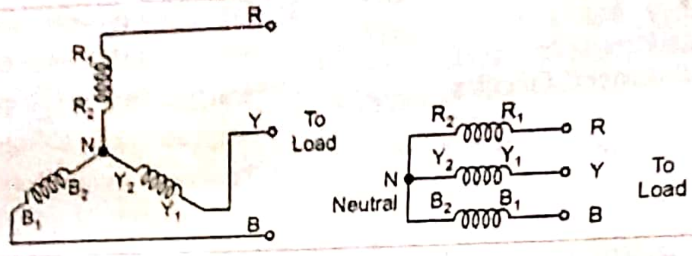

Fig. 2.23.1 Star connection

- The Fig. 2.23.2 shows the delta connection of supply lines.

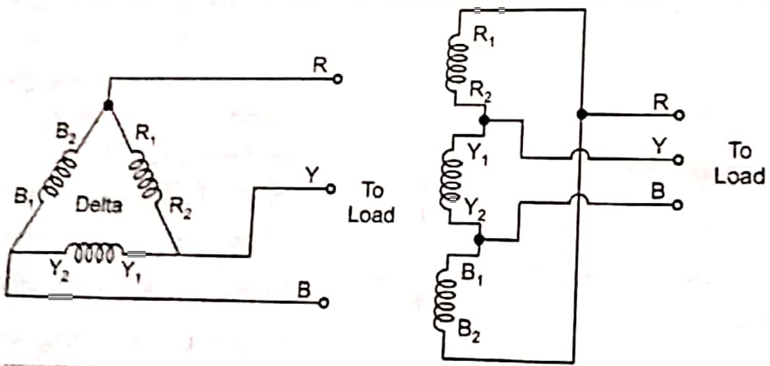

Fig. 2.23.2 Delta connection

2.24 : Line and Phase Quantities

Q.58 Define line and phase voltage and current for star and delta connections of load connected to three phase supply.

[JNTU : Part A, Dec.-09, May-11, Marks 3]

Ans. : The voltages across the supply lines are called line voltages denoted as V_L while the currents flowing through the lines connecting supply to the load are called line currents denoted as I_L. Thus V_{RY}, V_{YB}, V_{BR} are the line voltages while I_R, I_Y, I_B are the line currents.

- The voltage across any branch of the three phase load i.e. across Z_{ph1} or Z_{ph2} or Z_{ph3} is called phase voltage denoted as V_{ph}. The current passing through any branch of the three phase load is called phase current denoted as I_{ph}.

- The line and phase quantities for star load are shown in the Fig. Q.58.1(a) while the line and phase quantities for delta load are shown in the Fig. Q.58.1(b).

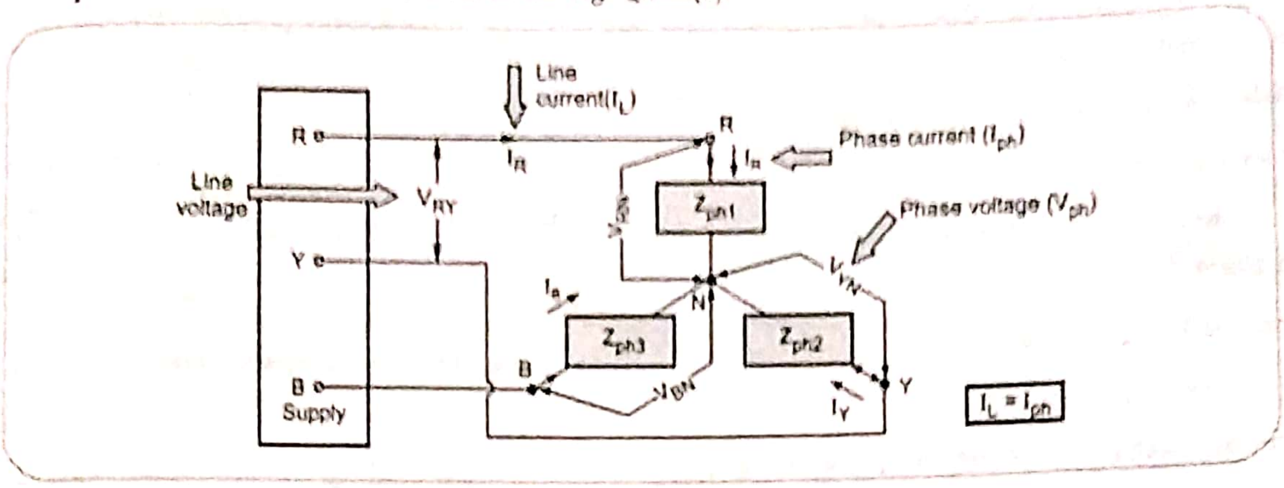

Fig. Q.58.1 Star load

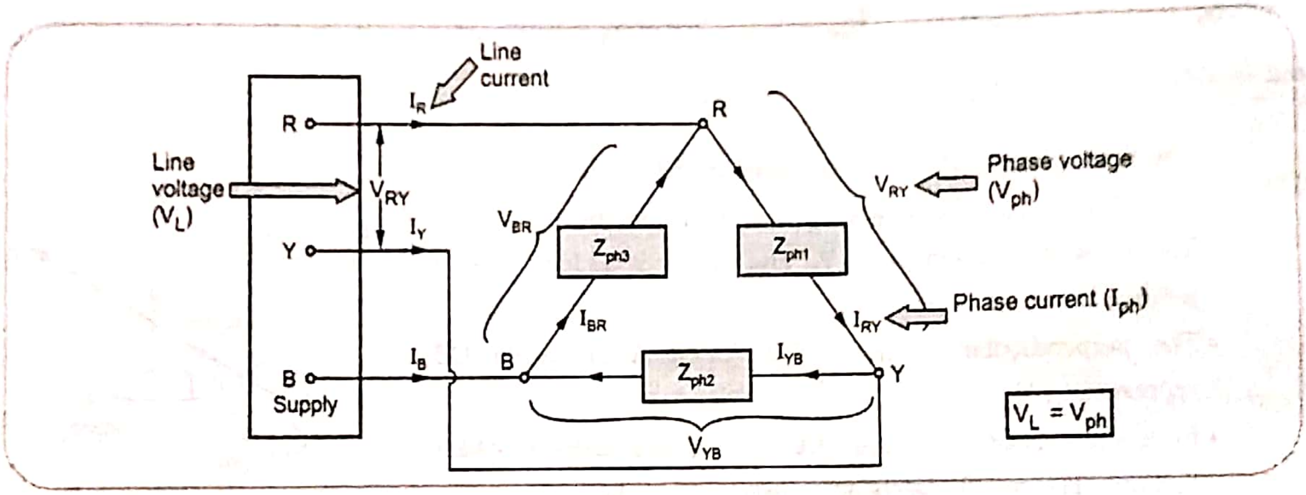

Fig. Q.58.2 Delta load

2.25 : Balanced and Unbalanced Load

Q.59 Define balanced and unbalanced three phase loads. [JNTU : Part A, Dec.-10, Marks 2]

Ans. : • The load is said to be balanced when magnitude of all the impedances Z_{ph1}, Z_{ph2} and Z_{ph3} are equal and the phase angles of all of them are equal and of same nature either all inductive or all capacitive or all resistive.

- The load is said to be unbalanced when magnitude of all the impedances Z_{ph1}, Z_{ph2} and Z_{ph3} are unequal and the phase angles of all of them are unequal.

2.26 : Relations for Star Connected Load

Q.60 Derive the relation between line and phase quantities in a three phase star connected circuit. Derive the expression for the power.

[JNTU : Part B, Dec.-18, Marks 5]

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Ans. : Consider the balanced star connected load as shown in the Fig. Q.60.1.

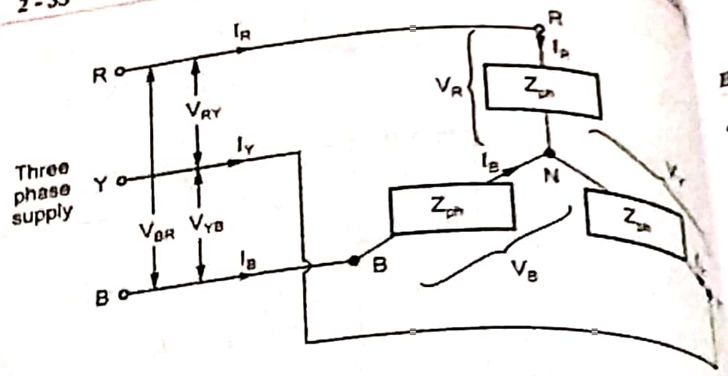

Fig. Q.60.1 Star connected load

- Line voltages, $V_L = V_{RY} = V_{YB} = V_{BR}$ and

  Line currents, $I_L = I_R = I_Y = I_B$
- Phase voltages, $V_{ph} = V_R = V_Y = V_B$ and

  Phase currents, $I_{ph} = I_R = I_Y = I_B$
- It can be seen that the line currents are same as the phase currents i.e. $I_L = I_{ph}$.
- From the Fig. Q.60.1 we can write,

$$\bar{V}_{RY} = \bar{V}_{RN} + \bar{V}_{NY} \quad \text{But} \quad \bar{V}_{NY} = -\bar{V}_{YN}$$

(Generally suffix N is not used for phase voltages)

Hence $\bar{V}_{RY} = \bar{V}_R - \bar{V}_Y$

Similarly, $\bar{V}_{YB} = \bar{V}_{YN} + \bar{V}_{NB} = \bar{V}_{YN} - \bar{V}_{BN} = \bar{V}_Y - \bar{V}_B$

and $\bar{V}_{BR} = \bar{V}_B - \bar{V}_R$

- The three phase voltage are displaced by 120° from each other.
- The phasor diagram to get V_{RY} is shown in the Fig. Q.60.2. The V_Y is reversed to get $-V_Y$ and then it is added to V_R to get V_{RY}.
- The perpendicular is drawn from point A on vector OB representing V_L.
- In triangle OAB, the sides OA and AB are same as phase voltages. Hence OB bisects angle between V_R and $-V_Y$.

$\therefore \quad \angle BOA = 30°$

- And perpendicular AC bisects the vector OB hence

$$OC = CB = \frac{V_L}{2}$$

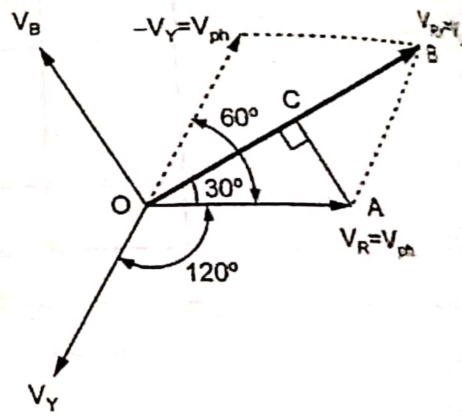

Fig. Q.60.2

- From triangle OAB,
- $\cos 30° = \frac{OC}{OA} = \frac{(V_{RY}/2)}{V_R}$ i.e. $\frac{\sqrt{3}}{2} = \frac{(V_L/2)}{V_{ph}}$

$V_L = \sqrt{3}\, V_{ph}$ for star connection and $I_L = I_{ph}$

Thus line voltage is $\sqrt{3}$ times the phase voltage in star connection.

- The lagging or leading nature of current depends on per phase impedance.

Power : The power consumed in each phase is single phase power given by,

$$P_{ph} = V_{ph}\, I_{ph} \cos \phi$$

- For balanced load, all phase powers are equal.

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- Hence total three phase power consumed is,

$$P = 3 P_{ph} = 3 V_{ph} I_{ph} \cos\phi$$

$$= 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi$$

$$\therefore \quad P = \sqrt{3} V_L I_L \cos\phi \quad \text{watts}$$

Q.61 Draw a complete phasor diagram for three phase star connected inductive load connected across three phase a.c. supply. **[JNTU : Part A, Marks 3]**

Ans. : The complete phasor diagram for lagging power factor star connected load is shown in the Fig. Q.61.1.

Each I_{ph} lags respective V_{ph} by angle ϕ

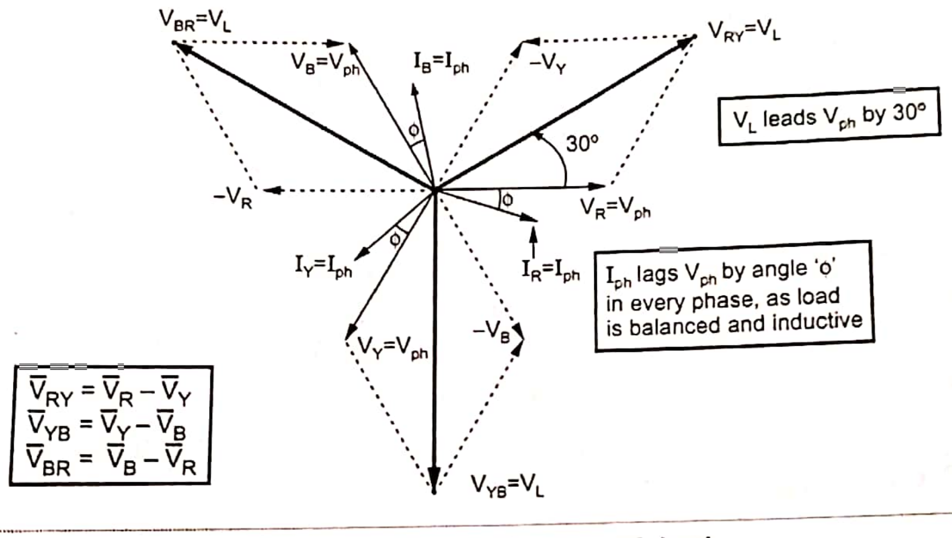

Fig. Q.61.1 Star and lagging p.f. load

2.27 : Relations for Delta Connected Load

Q.62 Derive the relation between line and phase quantities in a three phase delta connected circuit. Derive the expression for the power. **[JNTU : Part B, Marks 5]**

Ans. : Consider the balanced delta connected load as shown in the Fig. Q.62.1.

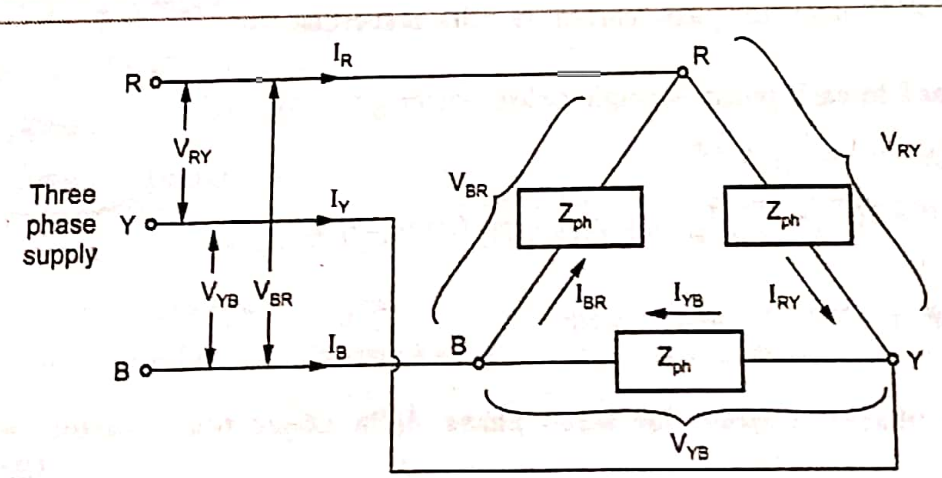

Fig. Q.62.1 Delta connected load

- Line voltages : $V_L = V_{RY} = V_{YB} = V_{BR}$ and
  Line currents : $I_L = I_R = I_Y = I_B$
- Phase voltages : $V_{ph} = V_{RY} = V_{YB} = V_{BR}$ and
  Phase currents : $I_{ph} = I_{RY} = I_{YB} = I_{BR}$
- It can be seen that the line voltages are same as the phase voltages i.e. $V_L = V_{ph}$.
- To derive the relation between I_L and I_{ph}, apply the KCL at the node R of the load shown in the Fig. Q.62.1.

$$\sum I_{incoming} = \sum I_{leaving} \text{ at node R}$$

$$\therefore \quad I_R + I_{BR} = I_{RY} \quad \text{i.e.} \quad I_R = I_{RY} - I_{BR} \quad ...(1)$$

Applying KCL at node Y and B, we can write equations for line currents I_Y and I_B as,

$$I_Y = I_{YB} - I_{RY} \text{ and } I_B = I_{BR} - I_{YB} \quad ...(2)$$

- The phasor diagram to obtain line current I_R by carrying out vector subtraction of phase currents I_{RY} and I_{YB} is shown in the Fig. Q.62.2.
- The three phase currents are displaced from each other by 120°.

I_{BR} is reversed to get $-I_{BR}$ and then added to I_{RY} to get I_R.

- The perpendicular AC drawn on vector OB, bisects the vector OB which represents I_L.
- Similarly OB bisects angle between $-I_{BR}$ and I_{RY} which is 60°.

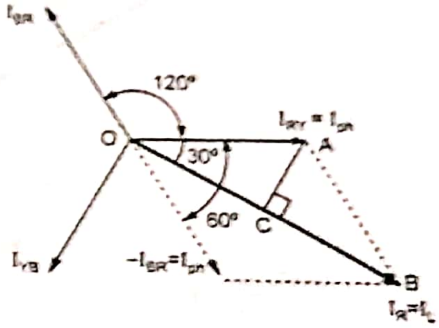

Fig. Q.62.2

$$\therefore \quad \angle BOA = 30° \quad \text{and} \quad OC = CB = \frac{I_L}{2}$$

- From triangle OAB, $\cos 30° = \frac{OC}{OA} = \frac{I_R/2}{I_{RY}}$ i.e. $\frac{\sqrt{3}}{2} = \frac{I_L/2}{I_{ph}}$

$$I_L = \sqrt{3}\ I_{ph} \text{ and } V_{ph} = V_L \quad \text{... for delta connection}$$

$\therefore$ **Thus line current is $\sqrt{3}$ times the phase current in delta connection.**

- Power consumed in each phase is single phase power given by,

$$P_{ph} = V_{ph}\ I_{ph} \cos\phi$$

- Total power $P = 3P_{ph} = 3V_{ph} I_{ph} \cos\phi = 3V_L \frac{I_L}{\sqrt{3}} \cos\phi$

$$\therefore \quad P = \sqrt{3}\ V_L I_L \cos\phi \quad \text{watts}$$

Q.63 Draw a complete phasor diagram for three phase delta connected inductive load connected acro three phase a.c. supply.

☞ [JNTU : Part A, Marks

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Ans. : The complete phasor diagram for lagging power factor delta connected load is shown in the Fig. Q.63.1

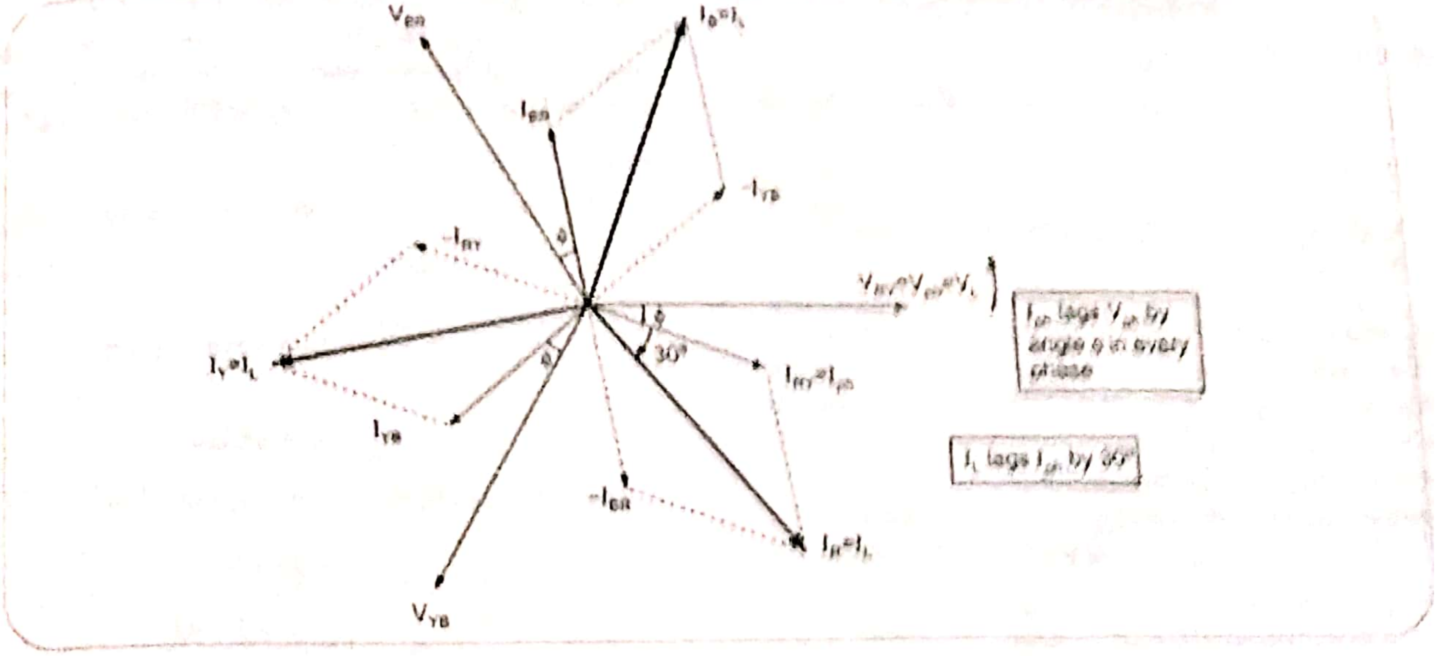

Fig. Q.63.1

> For delta connection, to draw phasor diagram use
> $\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}$, $\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY}$ and $\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB}$

- Each I_{ph} lags respective V_{ph} by angle ϕ

2.28 : Power Triangle for Three Phase Load

Q.64 State the equations for apparent power, active power and reactive power for three phase load. the power triangle. [JNTU : Part A, M

Ans. : • **Total apparent power,**

S = 3 × Apparent power per phase

$\therefore S = 3\ V_{ph} I_{ph}$

$= 3\ \dfrac{V_L}{\sqrt{3}} I_L = 3\ V_L \dfrac{I_L}{\sqrt{3}}$

$\therefore S = \sqrt{3}\, V_L I_L$ (VA) or (kVA)

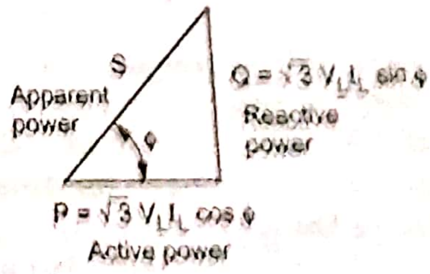

Fig. Q.64.1 Power triangle

- Total active power, $P = \sqrt{3}\, V_L I_L \cos\phi$ (W) or (kW)
- Total reactive power, $Q = \sqrt{3}\, V_L I_L \sin\phi$ (VAR) or (kVAR)
- The power triangle is shown in the Fig. Q.64.1.

Steps to Solve Problems on Three Phase Systems

1) Given supply voltages are always line voltages.
2) Determine phase voltage depending on whether load is star or delta connected.
3) Then determine phase current, $I_{ph} = \dfrac{V_{ph}}{Z_{ph}}$
4) Determine line current depending on whether load is star or delta connected.

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5) ϕ is angle between V_{ph} and I_{ph}. Value can be obtained from given Z_{ph}.

6) The total power consumed is $\sqrt{3}\, V_L I_L \cos\phi$.

7) If voltage and current are given then Z_{ph} can be obtained as $Z_{ph} = \frac{V_{ph}}{I_{ph}}$ and then comparing Z_{ph} with $R_{ph} \pm j X_{ph}$, the values of the elements of the load can be obtained.

Q.65 Three similar coils each having a resistance of 10 ohm and an inductance of 0.0318 H in series are connected in delta. The line voltage is 400 V, 50 Hz. Calculate :
i) phase current ii) line current
iii) power factor iv) total power in the circuit

[JNTU : Part B, Marks 5]

Ans. : $X_L = 2\pi f L = 2\pi \times 50 \times 0.0318 = 10\ \Omega$

$$\therefore \quad Z_{ph} = R + jX_L = 10 + j\,10 = 14.142\angle 45°\ \Omega$$

$$V_L = 400\ V,\ V_{ph} = V_L = 400\ V\ \text{(delta)}$$

i) $$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{14.142} = 28.284\ A\ \text{(magnitude)}$$

ii) $$I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 28.284 = 48.99\ A$$

iii) Power factor = cos 45° = **0.7071 lagging**

iv) $$P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 400 \times 48.99 \times 0.7071$$

$$= 24\ kW$$

Q.66 A 3 phase voltage source has a phase of 120 V and supplies star connected load having impedance of 24 + j36 Ω per phase. Calculate : i) Line voltage ii) Line current iii) Total 3-phase power supplied to the load.

[JNTU : Part B, Marks 5]

Ans. :

$$V_{ph} = 120\ V,$$

$$Z_{ph} = 24 + j\,36\ \Omega = 43.27\angle 56.31°\ \Omega$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{120}{43.27} = 2.773\ A\ \text{(magnitude)}$$

i) $V_L = \sqrt{3} V_{ph} = \sqrt{3} \times 120 = 207.846\ A$ ...(star)

ii) $I_L = I_{ph} = 2.773\ A$ (magnitude) ...(star)

iii) $$P = \sqrt{3} V_L I_L \cos\phi$$

$$= \sqrt{3} \times 207.846 \times 2.773 \times \cos(56.31°) = 553.65\ W.$$

Q.67 Three similar coils of impedance Z = (8+j6) Ω are connected in delta and supplied from 3φ 400 V, 50 Hz supply. Find line current, power factor, total active power, total reactive power, total volt amperes.

[JNTU : Part B, Marks 5]

Ans. :

$$Z_{ph} = 8 + j6\ \Omega = 10\angle 36.86°\ \Omega,\ V_L = 400\ V,\ \text{Delta}$$

$$V_{ph} = V_L = 400\ V$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{10} = 40\ A\ \text{(magnitude)}$$

$$\therefore \quad I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 40 = 69.282\ A$$

$$\cos\phi = \cos(36.86°) = 0.8\ \text{lagging}$$

$$P = \sqrt{3} V_L I_L \cos\phi = 38.4\ kW$$

$$Q = \sqrt{3} V_L I_L \sin\phi = 28.8\ kVAR$$

$$S = \sqrt{3} V_L I_L = 48\ kVA$$

Q.68 A balanced delta connected load impedance 16 + j12 Ω phase is connected to a three phase 400 V supply. Find the phase current, line current, power factor, active power, reactive power and total power. Also draw the phasor diagram.

[JNTU : Part B, Marks 5]

Ans. :

$$Z_{ph} = 16 + j\,12\ \Omega = 20\angle 51.34°\ \Omega,\ V_L = 400\ V$$

$$V_{ph} = V_L = 400\ V\ \text{for delta connected}$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{20} = 20\ A \quad \text{... Magnitude}$$

$$\therefore \quad I_L = \sqrt{3}\, I_{ph} = 34.641\ A \quad \text{... Magnitude}$$

The phasor diagram is as shown in the Fig. Q.63.1 of Q.63 with $\phi = 51.34°$.

$$\cos\phi = \cos(51.34°) = 0.6247\ \text{lagging}$$

$$P = \sqrt{3} V_L I_L \cos\phi = 14.993\ kW$$

$$Q = \sqrt{3} V_L I_L \sin\phi = 18.741\ kVAR$$

$$S = \sqrt{3} V_L I_L = 24\ kVA$$

Q.69 A balanced 3-phase star-connected load of 18 kW taking a leading current of 60 amperes when connected across a 3-phase 440 V, 50 Hz supply. Find the values and nature of load.

[JNTU : Part B, Marks 5]

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Ans. :

$P = 18$ kW, $I_L = 60$ A, $V_L = 440$ V, star, leading p.f.

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

i.e. $$18 \times 10^3 = \sqrt{3} \times 440 \times 60 \times \cos\phi$$

$$\cos\phi = 0.3936$$

i.e. $$\phi = -66.8183^\circ \quad \text{... -ve as leading}$$

$$|V_{ph}| = \frac{V_L}{\sqrt{3}} = 254.034 \text{ V},$$

$$|I_{ph}| = |I_L| = 60 \text{ A} \quad \text{... star load}$$

∴ $$|Z_{ph}| = \frac{|V_{ph}|}{|I_{ph}|} = \frac{254.034}{60} = 4.2339\ \Omega$$

∴ $$Z_{ph} = |Z_{ph}| \angle\phi = 4.2339 \angle -66.8183^\circ\ \Omega$$

$$= 1.667 - j3.892\ \Omega = R - jX_C$$

∴ $$R_{ph} = 1.667\ \Omega, \quad X_{Cph} = \frac{1}{2\pi f C_{ph}} = 3.892$$

∴ $$C_{ph} = \frac{1}{2\pi \times 50 \times 3.892}$$

$$= 0.8178 \text{ mF, Capacitive load.}$$

Fill in the Blanks for Mid Term Exam on Unit - I

Q.1 The voltage drop across 8 Ω resistance is ______ .

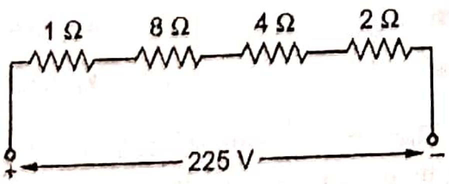

Fig. 1

Q.2 The resistance of a conductor is ______ proportional to its length. ☞ [JNTU : Oct.-16]

Q.3 Three 6 Ω resistors are connected in a delta network then the resistance value of each resistor, if it is converted in to a star network is ______. ☞ [JNTU : Oct.-16]

Q.4 If the voltage across a 25 ohms resistor is 5 V, the power dissipated by the resistor is ______. ☞ [JNTU : Aug.-16]

Q.5 If 1 A current flows in a circuit, the number of electrons flowing through this circuit is ______. ☞ [JNTU : Aug.-16]

Q.6 Abbrevation of KCL is ______ . ☞ [JNTU : Aug.-16]

Q.7 The equivalent resistance when a resistor of (1/3) Ω is connected in parallel with a (1/4) Ω resistance is ______ .

Q.8 The current 'x' in the Fig. Q.8.1 shown is ______ A.

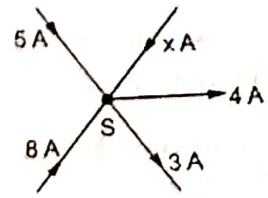

Fig. Q.8.1

Q.9 An alternating emf is given by $e = 200 \sin 314\, t$. The instantaneous value of emf at $t = 1/200$ sec is ______ .

Q.10 The peak factor of a sinusoidally varying voltage is ______ .

Q.11 An alternating current is $14.142 \sin(100\,\pi t - 30^\circ)$ A and an alternating voltage is $282.842 \sin(100\,\pi t + \frac{\pi}{4})$ V then the phase difference between V and I is ____ .

Q.12 The unit of admittance is ______ .

Q.13 If the circuit contains purely resistive elements then the power factor is ______.

Q.14 In a pure capacitive circuit, voltage lags by ______ the current.

Q.15 The admittance is a ______ of impedance.

Q.16 The product of voltage and current in an A.C. circuit gives ______ power.

Q.17 The active power P in a single phase circuit is given by ____ .

Q.18 An A.C. current is given by $I = 14.14 \sin(\omega t + 30^\circ)$ has an rms value of __ A .

Q.19 A sine wave has a frequency of 50 Hz its angular frequency is ___ rad/sec.

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